How do central banks control inflation?
A guide for the perplexed[*]

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Abstract

Central banks have a primary task of pursuing price stability. They do so by issuing different forms of money, setting an array of interest rates, producing fiscal revenues, defining the unit of account, and affecting marginal costs of production via credit regulations and other policies. This article surveys the economic theories that justify the central bank's ability to use these tools to control inflation around a target. It presents alternative approaches as consistent with each other, as opposed to as conflicting ideological camps. Each of them relies on equilibrium forces in different markets within a common dynamic general equilibrium structure.

JEL codes: E31, E52, E61.

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Outline

1 Introduction

How do central banks keep inflation on target? How do they prevent episodes of hyper-inflation and their tragic consequences for welfare? Can the central bank control inflation if the economy goes through secular stagnation, a liquidity trap, or a fiscal crisis? These are crucial questions, both for policymakers and for academics, and they have answers in current economic theory.

Yet, students coming out of a macroeconomics class are often flummoxed by this topic. Undergraduates mostly retain that central banks print money and more money means higher inflation. They are then thoroughly confused when they realize that most central banks barely mention money in their speeches, that they do not actually choose how much money to print, and that the US monetary base increased five-fold between 2008 and 2014 with no visible dent on inflation. Graduate students learn about the setting of interest rates and the Phillips curve, and perhaps even about the welfare costs of inflation and the links between monetary and fiscal policy. However, as soon as you ask them to reconcile the Fisher principle (higher interest rates are associated with higher expected inflation one-to-one), the Taylor rule (increasing interest rates more than one-to-one in response to inflation keeps inflation constant), and the empirical evidence (raising interest rates lowers inflation) you are likely to get an incoherent answer. Discussions on equilibrium determinacy or active-passive regimes attract theoretically-minded researchers as much as they put off those focused on empirical applications.

The goal of this article is to provide a unified treatment of the theory of how central banks control inflation. The hope is that researchers will have an accessible entry point to this literature, so they can make sense of monetary policies and inflation outcomes using current economic theory. Our approach is to highlight the common features of different viewpoints by using a single neoclassical general-equilibrium model of the economy, and arguing that alternative theories simply focus on different equations and markets within the same model. The central bank is at the center of all of them, as the agency with a mandate to deliver a value for the price level, although policies can be described as monetary, fiscal or, more accurately, as a mix of both.

This is not an article about the optimal way to conduct monetary policy or about how to trade off variability in inflation versus real activity. We take as given a target for inflation, and study how the central bank goes about delivering actual inflation as close as possible to this target. This involves two questions. First is a *determinacy* question, on

whether policy can deliver a unique price level. Multiple or indeterminate equilibria in models are signs of outcomes in the real world that may be quite far from what policy intended. Second, focusing on which policy leads to a smaller variation in the deviations of actual inflation from target, we answer an *effectiveness* question. By characterizing the components of this variation so that they can be quantified, the choice between monetary policies can rely less on the aesthetic beauty of their theories and more on their predicted outcomes. Inversely, different historical experiences with inflation are interpreted as different policy approaches being in use, as opposed to evidence for or against one of them.

Our framework is organized in three levels. Each section presents an *approach* to control inflation. This is understood as a focus on a particular market, associated with a specific equilibrium condition, and an equilibrating economic force that moves the price level. Each section also introduces one or more *tools* within the toolkit of the central bank that directly affect one market to steer inflation towards its target. Finally, within each approach-tool mix, there are different *rules* that link the tool to exogenous policy choices. We also discuss the most effective rule in order to compare different approaches. Towards the end of the paper, we bring all the approaches together to show that in practice they can all work at the same time, but policy must choose which one is the active one driving inflation.

Before we get started, section 2 sets up the canonical dynamic model we will use and shows that the classical analysis of supply and demand does not pin down inflation. This is what makes this topic special. The section also provides a description of what is a minimal central bank. Each section that follows expands on it as it introduces more tools available to the central bank.

In section 3, we consider the approach that focuses on financial markets and the forces of arbitrage that price financial assets. The key equilibrium condition is the Fisher equation, although terminal conditions on nominal variables, regime switches, and the rationality of expectations, also play a role. The associated policy tool that the central bank chooses is the interest rate, either the one observed in short-term bonds, or the one that is set by remunerating bank deposits at the central bank. The policy rules include conventional feedback rules for short-term rates, like the Taylor rule, as well as unconventional policies, such as going long on interest rates through forward guidance or offering a fixed real payment on reserves.

Section 4 looks at the market for currency and how the price level may adjust to ensure it is in equilibrium. The key equation is the demand for currency, while the policy tool

is the monopoly power of the central bank to print banknotes. We consider both money growth rules, as well as fiscal rules on seignorage revenues from this activity.

In section 5, the key equation is an intertemporal budget constraint. The approach in this section relies on the solvency of the central bank or, in some cases, on the solvency of the overall government. The price level needs to adjust to rule out Ponzi schemes on government debt. This is usually called a fiscal-theory of price level determination. But it can be employed by a central bank using as its tool the surplus it earns by varying the size and composition of its balance sheet. Rules for the central bank's dividends or its net worth steer inflation.

In section 6, we focus on the market for goods and on the law of one price as the economic force that pins down inflation. The central bank policy tool here is its power to define the unit of account. Because many of the goods consumed in a country are produced in another country with a different unit of account, this often involves rules for exchange rate pegs.

The final approach looks at the Phillips curve as the key economic relation. Inflation is determined via the link between real outcomes and nominal variables. The central bank uses credit regulation as well as all of the tools discussed so far to affect marginal costs of firms and to control inflation. While the previous four sections assumed flexible prices to keep the analysis simple, section 7 discusses how earlier conclusions might change with nominal rigidities (answer: not much).

Arbitrage, supply and demand, solvency, the law of one price, and nominal rigidities are five separate economic forces that co-exist in a well-specified model where they can interact with each other. Section 8 brings together the equations highlighted in each of the previous sections to conclude. This makes clear that alternative theories of inflation are rather different approaches that central banks choose to follow. Importantly, these choices have to be consistent with each other, requiring that one policy tool is dominant over the other.[1]

2 Inflation in equilibrium

At the core of most dynamic macroeconomic models is an Euler equation of the form:

$$\mathbb{E}_t \left[M_{t+1}(1 + R_t) \right] = 1. \tag{1}$$

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[1]Previous surveys, taking a different approach, are McCallum (1999) and Woodford (2003).

The $\mathbb{E}_t(.)$ operator captures the rational expectations of the private sector as of date t. R_t is the promised return at date t on a real safe investment that pays off at date $t + 1$, while M_{t+1} is a stochastic discount factor.

Intuitively, M_{t+1} reveals how many units of a good the private agents would require next period in exchange for one unit of the good today. In other words, M_{t+1} is the marginal rate of substitution between consumption today and tomorrow. Since $1 + R_t$ is the opportunity cost of consuming one more unit today in terms of foregone consumption tomorrow, then the equation above is the standard optimality condition stating that agents must be indifferent between consuming or saving an extra unit.

An alternative, but equivalent, investment intuition is that to ensure no-arbitrage profits, it must be that the risk and time adjusted net return on any investment is zero. The stochastic discount factor provides the adjustment factor for time and risk. If investors are risk neutral then M_{t+1} would be equal to a constant β that captures solely impatience, and the equation states that the real return is approximately equal to the rate of time preference $- \ln(\beta)$.

A stark assumption that we will keep until section 7 is that M_{t+1} is exogenous with respect to P_t. This assumption is commonly known as the classical dichotomy. It states that real trade-offs are unchanged regardless of the price level. An *equilibrium* in this economy is a solution for $\{R_t, P_t\}_{t=0}^{\infty}$ such that given an exogenous $\{M_{t+1}\}_{t=0}^{\infty}$, equation (1) holds.

2.1   Price level (in)determinacy

In equilibrium, the real interest rate is given by:

$$R_t = \mathbb{E}_t \left[M_{t+1}\right]^{-1} - 1, \tag{2}$$

at all dates. Nothing pins down the price level. Any sequence $\{P_t\}_{t=0}^{\infty}$ is consistent with the Euler equation and the economy being in equilibrium. Setting $\{P_t\}_{t=0}^{\infty}$ merely amounts to setting the unit of account in the economy.

More formally, let s_t be the state of the world at date $t \geq 1$, and let $s^t = (s_1, ..., s_t)$ be the history of states until date t. Define inflation as $\Pi_t(P_0, s^t) \equiv P_t / P_{t-1}$, the change in the price level. A nominal equilibrium is then an initial value P_0 and a function $\Pi_t(P_0, s^t)$ for all dates $t \geq 1$.

Definition. *The level of inflation is unique or determinate in equilibrium if:*

1. *There is a unique scalar P_0 in equilibrium.*

2. *If $\Pi'_t(P_0, s^t)$ and $\Pi''_t(P_0, s^t)$ both satisfy equilibrium conditions, then $\Pi'_t(P_0, s^t) = \Pi''_t(P_0, s^t)$.*

The first condition requires that even if the entire future path of inflation is pinned down from today onward, one must still know today's price level. What ultimately matters is how much is a dollar's worth in terms of real goods at any given date. Without pinning down the initial value of a dollar, for a given inflation path, the actual price level P_t could be any number.

The second condition states that, for any given state of the world, inflation must be unique. If, in spite of all the fundamental features of the world being the same, inflation can be different, then the central bank has failed to pin down inflation.

The result that, without a central bank, inflation is indeterminate in equilibrium holds in *any* classical model. The argument dates back to Hume (1752): dollars are just a unit of account with which the prices of goods are determined. If people started denominating prices in cents instead of dollars nothing would change. There is no demand or supply that ensures that 100 cents equals one dollar. Nothing in classical economics pins down the price level or inflation, in the same way that nothing determines whether measurements should be in inches or centimeters.

2.2   Nominal bonds do not solve indeterminacy

A natural reaction to this result would be to say that since the price level does not appear in the Euler equation, of course it cannot be determinate. If savings are in real assets, nothing is denominated in nominal units within the model.

Introduce now nominal bonds. The optimality condition with respect to holdings of these bonds is:

$$\mathbb{E}_t \left[M_{t+1} \left(\frac{1 + I_t}{\Pi_{t+1}} \right) \right] = 1. \tag{3}$$

As nominal bonds promise a nominal interest rate I_t, their real return depends on inflation. Indifference towards holding them must result from equating this expected return times the marginal rate of substitution to one.

While this gives an additional equilibrium condition, there is also an additional endogenous variable, I_t. The above equation pins down I_t given Π_t. But there is still nothing to pin down Π_t in the first place. Introducing nominal bonds does not solve price level indeterminacy.

2.3 Introducing a central bank, its aims and tools

The *policy aim* of the central bank is to keep $\{P_t\}_{t=0}^{\infty}$ close to a target $\{P_t^*\}_{t=0}^{\infty}$. The target may be stochastic, have a unit root, or depend on the real state of the economy captured by M_t. It may be arbitrary or optimal given some objectives of policy.[2] The key assumption is that it is exogenous with respect to P_t.

To achieve this target, central banks have different *policy tools*. We start by describing a minimal central bank, and throughout the paper expand on it to include the different roles that central banks perform around the world.

In modern digital economies, people use electronic means of payment like debit or credit cards to settle their transactions. For any given transaction, the seller may have an account in bank A and the buyer an account in bank B, so there must be a settlement whereby bank A collects payments from bank B. The central bank is the clearing house where payments between banks take place. These payments are made using another digital mean of payment, often named reserves, which are nothing but liabilities of the central bank towards banks.[3] Because reserves are the ultimate form of payment, they are the unit of account. Since reserves in the United States are denominated in dollars, firms and people choose to denominate their prices in dollars as well. The price of a good is simply how many units of reserves must be exchanged to obtain the good.

The current stock of reserves is just a list of entries in a spreadsheet at the central bank, one for each bank. Given its control over the spreadsheet, the central bank has two ways to set monetary policy: it can choose the amount of reserves, V_t, or the rate at which it remunerates them, I_t^v. These are nothing but decisions on the sum of the entries in the spreadsheet at any point in time, and on the rate at which the number in each entry of the spreadsheet rises across periods.

The policy tools are set according to *policy rules*. The central bank can choose a peg for the interest on reserves, which does not depend on any other economic variable, or it can make the interest rate depend on the state of the world driving the real interest rate or on the inflation target. We denote this choice by X_t^i, to highlight that it is exogenous with respect to P_t, and with the superscript denoting the tool it refers to. The policy rule may also imply a feedback from the actual price level to the policy tool, in which case the

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[2]Readers interested in the choice of P_t^* can see Khan, King and Wolman (2003) or Woodford (2010).

[3]For people that prefer to settle some transactions without using a bank (a minority today) the central bank also issues banknotes, a particular durable good, and commits to exchange them for reserves one-to-one at all times. We will discuss currency in section 4.

policy rule is a map from P_t and X_t^i to the tool I_t^v.

2.4 A measure of effectiveness

To study the effectiveness of a rule, we log-linearize the economy around a steady state point where the real interest rate and inflation are equal to constants, β and Π. Small letters denote the log deviations from that constant.

Definition. *The effectiveness of a policy is assessed by how small the sequence of deviations between the log price level and its target is:*

$$\varepsilon_t \equiv p_t - p_t^*. \tag{4}$$

Given an approach to controlling inflation, the central bank would like to set its tool following a rule that makes its expectation of these errors zero. We call this the *most effective* rule, and denote its choice by X_t^*.

However, the central bank only has imperfect real-time estimates of the real interest rate or of the desired inflation target. To clarify the notation $\mathbb{E}_t(p_{t+j})$ is the public's expectation at t of what the price level will be at date $t + j$, while $\hat{p}_{t+j}$ is the central bank's expectation at t, and these may not be the same. The most effective rule is therefore defined by ensuring that in equilibrium $\hat{p}_t = \hat{p}_t^*$.

The remainder of this paper introduces different tools available to central banks, and judges them in terms of the determinacy of inflation and the effectiveness of the rules they inspire. Before proceeding, we clarify the micro-foundations of the model.

2.5 General equilibrium micro-foundations

For readers uncomfortable with just stating equations (1) and (3), here is a simple micro-foundation. Consider an exchange economy populated by many private agents that have the same time-separable preferences over a single consumption good $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$ and trade real bonds K_t and nominal bonds B_t with each other subject to a budget constraint: $P_t C_t + P_t K_t + B_t \leq P_t Y_t + P_t K_{t-1}(1 + R_{t-1}) + B_t(1 + I_{t-1})$. There is no storage technology and the bonds are in zero net supply, so market clearing imposes that consumption is equal to aggregate output $C_t = Y_t$, which is a random endowment. Optimal behavior in this economy is then entirely described by equations (1) and (3) where the

stochastic discount factor is:

$$M_{t+1} = \beta U'(Y_{t+1})/U'(Y_t). \tag{5}$$

No model of inflation is complete without specifying how the central bank interacts with the fiscal authority. In this minimal model, government bonds were the negative of reserves outstanding so that nominal liabilities were issued in zero supply. The level of V_t is irrelevant, since reserves and other liabilities of the government have no effect on any real or nominal outcome by Ricardian equivalence. Likewise, the minimal central bank earns no income and rebates no dividends to the government. The fiscal authority may have spending but it collects its own taxes to pay for it. This is sometimes called a Ricardian policy. Later, in section 5, we discuss an alternative where the quantity of reserves can be used to control inflation and where the dividends of the central bank become important.

3 The no-arbitrage approach: setting interest rates

The most common approach followed by central banks for decades is to fix the nominal interest rate of one-period bonds I_t. In the past, this was done indirectly by rationing the supply of reserves, and then buying and selling bonds for reserves to target this interest rate. Over the last decade, the nominal interest rate was set directly by remunerating reserves at the rate I_t^v, and having abundant reserves so that they are a pure financial asset that provides no payment or liquidity services. In that case, no arbitrage between nominal bonds and reserves delivers $I_t = I_t^v$.

In turn, the no-arbitrage relation between real and nominal bonds is:

$$\mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{1 + I_t}{\Pi_{t+1}} \right) \right] = 0. \tag{6}$$

This states that, once adjusted by the stochastic discount factor, savings in real investment or in reserves at the central banks must yield the same expected return. It is often called the Fisher equation and it is the key equation of the approach in this section.

The Fisher equation captures an economic force that can move the price level. It works as follows: banks can choose to hold reserves or real investments. Suppose the price level today was too low. Relative to a fixed future price level, then expected inflation would be

higher. Therefore, the return on reserves would be lower than that on real investments. In other words, by holding reserves at the central bank, banks get fewer goods in return than if they had invested them privately. Banks would want to hold zero reserves and invest all of their resources in real terms, which would not be an equilibrium given a positive supply of reserves. Rather, as banks demand fewer reserves, their value falls. Because reserves are the unit of account, their real value is $1/P_t$, so the price level must rise back into equilibrium. A higher price level means that expected inflation is lower and the real return on reserves is higher, rising until the point where banks are, once again, indifferent between real investment and reserves.[4]

3.1 Interest rate pegs

If the central bank chooses the interest on reserves exogenously, then $I_t = I_t^v = X_t^i$. The literature has traditionally referred to this as an interest rate peg. The Fisher equation then implies that:

$$\mathbb{E}_t \left(\frac{M_{t+1}}{\Pi_{t+1}} \right) = \frac{1}{1 + X_t^i}. \tag{7}$$

By choosing the right-hand side, the minimal central bank is able to pin down the expected ratio of the stochastic discount factor and inflation. Inflation itself though is not determinate. There are an infinite number of inflation rates at different states of the world that satisfy this equation.[5]

If there is no uncertainty in the economy, the expectations operator disappears from equation (7). In that case, by choosing X_t^i, the central bank ensures a single Π_{t+1} at each date. Even then, there is no other condition to pin down P_0. If people expect higher prices in the future, the price level at 0 will simply jump up today, keeping inflation equal to $(1 + X_t^i)/(1 + R_t)$. The first condition for determinacy is not satisfied.[6]

By itself, relying on arbitrage and setting interest rates does not determine inflation.

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[4]A small literature has studied inflation using the no-arbitrage approach but when incomplete markets lead to variations of equation (6). In the interest of space, we do not cover these, although the economic logic is similar to the one we present. See Benassy (2000), Den Haan, Rendahl and Riegler (2017), or Hagedorn (2018).

[5]See Nakajima and Polemarchakis (2005) for a thorough discussion across different economic environments.

[6]This classic result is due to Sargent and Wallace (1975).

3.2 Interest rate feedback rules

The central bank can do more. It can choose a feedback rule so that it adjusts the interest rate in response to contemporaneous movements in the price level (or inflation).

3.2.1 The Taylor rule determines inflation

The most famous of these rules is:

$$i_t = x_t^i + \phi \pi_t. \tag{8}$$

The Taylor principle sets $\phi > 1$, so the response to inflation is more than one-to-one. An enormous literature has used this rule for monetary policy, following Taylor (1993)'s demonstration that it fits the U.S. experience well.

The log-linearized version of the Fisher equation is:

$$i_t = r_t + \mathbb{E}_t(\pi_{t+1}). \tag{9}$$

Combining it with the Taylor rule to replace out the nominal interest rate delivers a difference equation for the deviations of inflation from target. Iterating forwards and imposing a terminal condition, such that $\lim_{T \to \infty} \phi^{-T} \mathbb{E}_t \left(\pi_{t+T} - \pi_{t+T}^* \right) = 0$, delivers a unique solution:[7]

$$\pi_t = \pi_t^* + \sum_{j=0}^{\infty} \phi^{-j-1} \mathbb{E}_t \left(r_{t+j} + \pi_{t+1+j}^* - \phi \pi_{t+j}^* - x_{t+j}^i \right). \tag{10}$$

Note that this equation holds for all $t \geq 0$. Since P_{-1} is given, the price level is determinate at all dates, including 0.

The most effective policy rule sets the interest rate to respond to inflation as well as to the central bank's forecast of real interest rates and the inflation target: $x_t^{i*} = \hat{r}_t + \hat{\pi}_{t+1}^* - \phi \hat{\pi}_t^*$. Its effectiveness is:

$$\varepsilon_t = \varepsilon_{t-1} + \sum_{j=0}^{\infty} \phi^{-j-1} \mathbb{E}_t \left[r_{t+j} - \hat{r}_{t+j} + \pi_{t+1+j}^* - \hat{\pi}_{t+1+j}^* - \phi(\pi_{t+j}^* - \hat{\pi}_{t+j}^*) \right]. \tag{11}$$

The right-hand side summarizes the public's expectations of the estimation mistakes

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[7]This equation also implicitly assumes that the infinite sum of expectations is finite. This is a weak requirement on policy: it must be sufficiently effective so that x_t^i does not stray away from the real interest rates and inflation exponentially over time.

made by the central bank on the state of the economy. Even if neither the central bank nor the public know what r_t or π_t^* are, and even if their estimates are poor, as long as these estimates coincide, the policy will be effective.

The literature calls Delphic forward guidance to central bank communications of what it thinks the future states of the economy will be. With Taylor rules, this is a crucial part of policymaking. It is more important for effectiveness that the central bank's estimates coincide with those of the public, than it is for them to be right.

3.2.2 Other interest rate feedback rules may be more effective

A large literature has discussed the general class of interest rate rules and estimated them using data across countries and time regimes.[8] Many of them fit into the formulation in equation (8). For instance, a wide variety of measures and estimates of real activity have been included in the rule, to capture the fact that central banks often respond to recessions by cutting interest rates. All of these measures fall under the x_t^i term in our notation. Since our analysis already allowed for (r_t, x_t^i) to be general stochastic processes—the only restriction was that they were exogenous with respect to inflation—we have already covered their determinacy. They only affect the effectiveness of the policy.

We can also consider a few broader classes of feedback rules. First, most estimates of policy rules also show that interest rates are inertial. Central banks typically break a desired change in interest rates into 0.25% steps over successive policy meetings. We can represent this by having current interest rates responding to their own past value. Second, convinced by estimates that monetary policy only affects inflation with a lag, many central banks adjust interest rates in response to public forecasts of future inflation, obtained from surveys, financial prices or internal models of the central bank. We capture this by adding the public's expectation of future inflation to the interest rate rule. Third, many central banks respond to core measures that try to smooth out the noisy real-time measures of inflation and capture its permanent trends. Following Muth (1960), we model core inflation as a weighted average of past inflation, which is the optimal estimate if actual inflation follows a random walk contaminated with white noise measurement error. Fourth, studies of optimal monetary policy often suggest that the central bank should target the price level rather than inflation. These Wicksellian rules replace π_t with p_t in the policy rule.

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[8]McCallum (1981) introduced these rules and first showed that they lead to determinacy. Clarida, Galí and Gertler (2000) and Woodford (2003) are classic analyses.

Table 1: Feedback rules and determinacy conditions

Name	Rule	Determinacy condition
Benchmark	$i_t = x_t^i + \phi \pi_t$	$\phi > 1$
Inertia	$i_t = x_t^i + \phi \pi_t + \chi i_{t-1}$	$\phi + \chi > 1$
Forecast targeting	$i_t = x_t^i + \phi \pi_t + \chi \, \mathbb{E}_t(\pi_{t+1})$	$\phi + \chi > 1$
Core inflation	$i_t = x_t^i + \phi(1 - \chi) \sum_{j=0}^{\infty} \chi^j \pi_{t-j}$	$\phi > 1$
Wicksellian rule	$i_t = x_t^i + \phi p_t$	$\phi > 0$

The mathematics and economic logic of all these cases are similar to the ones in the analysis of the Taylor rule. Table 1 formalizes them and shows the determinacy conditions derived from the same steps as in the previous section. In all of them, the response of interest rates to inflation must be large enough, although the thresholds differ, as shown in the last column of the table. Also, all the formulas for effectiveness depend on the ability of the central bank to minimize the discrepancy between the public's and the central bank's forecasts of the state of the economy and the value of the inflation target. Depending on the relative variances and correlation between the state of the economy and the inflation target, some rules will be more effective than others.

3.2.3 The economic force behind feedback rules

If $\varepsilon_t = 0$, the Taylor rule (or any of the other feedback rules) is observationally equivalent to the peg of the previous subsection, since then $\pi_t = \pi_t^*$, and all that one observes is $i_t = x_t^i + \pi_t^*$, an exogenous process. Yet, the mere presence of $\phi > 1$ solves the two degrees of indeterminacy discussed in the previous section.[9] How?

Imagine that inflation is higher at date t by one log unit relative to the solution above. Then, with the Taylor rule the central bank will raise the nominal interest rate by ϕ leading to an increase in expected inflation between t and $t+1$ of ϕ (the logic is the same for the other rules). But this in turn leads the central bank to raise i_{t+1} by ϕ^2, which raises expected inflation between $t+1$ and $t+2$ by that amount. The process continues so inflation keeps on rising exponentially and the feedback rule imposes inflation in T periods to be larger by ϕ^T. As the terminal condition rules out these deviations, the inflation target level is the unique possible solution. But where does the terminal condition come from in the first place?

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[9]As a result, data on inflation and interest rates by itself cannot provide an estimate of ϕ (Lubik and Schorfheide, 2004; Cochrane, 2011).

3.2.4 The elusive terminal condition

With the Taylor rule, the terminal condition is:

$$\lim_{T \to \infty} \phi^{-T} \, \mathbb{E}_t \left(\pi_{t+T} - \pi^*_{t+T} \right) = 0. \tag{12}$$

Equivalently, the random variable $\mathbb{E}_t \left(\pi_{t+T} - \pi^*_{t+T} \right)$ belongs to $O(\ln(\phi))$. That is, if expected inflation deviates from target, those deviations cannot grow faster than at the rate $\ln(\phi)$. The larger is ϕ, the weaker is this condition.

The terminal condition is not an optimality condition, the way that transversality conditions are. Those apply to the real value of savings, whereas the condition needed here is on a purely nominal variable, the price level. Additionally, optimal behavior imposes no money illusion in the Euler equation or in the transversality conditions. Furthermore, there is no sense in which the economy blows up if this condition does not hold. The unit of account may be exploding, but agents with no money illusion would be indifferent as real outcomes and variables continue to be finite.

The most common justification for the terminal condition is that the equilibria that violate it are not plausible. The feedback rule ensures that any of these equilibria associated with indeterminacy leads to explosive paths for inflation. Perhaps people would never believe them. More formally, if people's expectations of inflation deviations from target in the future are constrained to stay locally bounded, then $\mathbb{E}_t \left(\pi_{t+T} - \pi^*_{t+T} \right)$ is $O(0)$ and the Taylor principle implies the terminal condition. Among the set of bounded equilibria, inflation is determined.

A related argument notes that since the derivations above relied on log-linearization of the Fisher equation, inflation should be bounded for the error to be small in this local approximation. Restricting attention to bounded equilibrium is coherent with how the model is being solved.[10]

3.3 Interest rate rules do not rely on the terminal condition

A recent literature has shown that unconventional ways of setting interest rates provide alternatives to feedback rules that do not make use of the elusive terminal condition, even if they still rely on no arbitrage.

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[10]Cochrane (2011) makes a scathing critique of these arguments.

3.3.1 Real payments on reserves

Imagine the central bank promises to remunerate reserve holders with a payment in real goods.[11] Governments have issued indexed bonds for a long time across the world, and so could central banks, and this is what promising a real payment of goods amounts to. The nominal return on reserves in dollars would then be $1 + I^v_{t,t+1} = (1 + X^i_t)P_{t+1}$.

Rearranging equation (6) and using the result in equation (2) then delivers:

$$\mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{(1 + X^i_t)P_{t+1}P_t}{P_{t+1}} \right) \right] = 0 \;\Leftrightarrow\; P_t = \frac{1 + R_t}{1 + X^i_t}. \tag{13}$$

Since X^i_t is exogenously chosen by policy, and R_t is exogenously pinned down by real forces, then the above equation delivers a determinate price level.

The intuition for how the price level is pinned down is the following. The real return on any investment is pinned down exogenously by the stochastic discount factor. If the central bank promises a real payment on reserves, then arbitrage pins down how many goods reserves are worth today. This is the economic force behind the Fisher equation: since real bonds and reserves both deliver the same payment tomorrow, they must be worth the same today. But, since reserves are denominated in dollars, not goods, then this pins down the price level today.

The rule $1 + X^{i*}_t = (1 + \hat{R}_t)/\hat{P}^*_t$ gives the control error:

$$\varepsilon_t = r_t - \hat{r}_t + \hat{p}^*_t - p^*_t. \tag{14}$$

The better the estimates of the real interest rate, the more effective this policy will be. Under this rule for monetary policy, only current estimation errors matter, not the sum of errors into the infinite future as it was the case under feedback rules.

3.3.2 Going long: setting long-term interest rates

Over the last decade, central banks went long in the sense that the focus of monetary policy became long-term interest rates. The Bank of Japan went the furthest by announcing a desired target for the 10-year interest rate, standing ready to buy and sell government bonds of this maturity to hit the target. Other central banks have instead used two forms

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[11]This was studied by Hall and Reis (2016), building on earlier work by Hall (1997), which in turn was an interpretation of Irving Fisher's proposal.

of unconventional monetary policy: announcing the path for future short-term interest rates (Odyssean forward guidance), and purchases of long-term bonds funded by issuing reserves (quantitative easing). Through these actions they imperfectly targeted long rates.

In theory, if the central bank issued bonds of a fixed maturity that were later paid off with reserves, it could choose how to remunerate these bonds just as it does with reserves. If the central bank issues a j period bond and pays I_t^j interest rate on it, then the Euler equation that applies to this new form of investment is:

$$\mathbb{E}_t \left[\frac{M_{t,t+j}(1 + I_t^j)}{\Pi_{t+1}\Pi_{t+2}...\Pi_{t+j}} \right] = 1. \tag{15}$$

The stochastic discount factor between two non-successive dates is: $M_{t,t+j} = M_{t+1}M_{t+2}...M_{t+j}$. By choosing a feedback rule for I_t^j in much the same way as it did for one-period reserves, the central bank can control the price level. The condition for determinacy still requires ϕ to be larger than some threshold, but the threshold is now equal to the sensitivity of long rates to short rates. The effectiveness of this policy involves similar terms but with different weights.[12]

3.3.3 Setting both short-term and long-term rates

Alternatively, the central bank may choose short-term and long-term interest rates simultaneously. In this case, the Euler condition provides an extra set of equations, one for each date t. Increasing the number of equations without increasing the number of unknowns gives hope that perhaps inflation is now determinate.[13]

To see this at play, consider the simple case in which there is only uncertainty about M_{t+1}, which follows a two-state stationary Markov chain with values M_H and M_L and transition matrix with non-negative probabilities satisfying $f_{HH} + f_{HL} = 1$ and $f_{LH} + f_{LL} = 1$. Controlling inflation boils down to determining the two values of inflation, Π_H and Π_L, uniquely. The Euler equations with respect to the one-period reserves and the

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[12]See McGough, Rudebusch and Williams (2005) and Reis (2019*a*) for derivations.

[13]This point was made by Adão, Correia and Teles (2014) and Magill and Quinzii (2014).

two-period bonds can be written at state s as:

$$(1 + I_s^1) \left(f_{sH} \frac{M_H}{\Pi_H} + f_{sL} \frac{M_L}{\Pi_L} \right) = 1, \tag{16}$$

$$(1 + I_s^2) \left(f_{sH} \frac{M_H}{\Pi_H(1 + I_H^1)} + f_{sL} \frac{M_L}{\Pi_L(1 + I_L^1)} \right) = 1. \tag{17}$$

These are two equations in two unknowns. Standard linear algebra shows that as long as $I_H^1 \neq I_L^1$, then there is a unique solution for inflation. The key condition for determinacy is now that the central bank does not set the interest on reserves to be the same across states of the world.

Note that this approach does not pin down P_0. Only the stochastic degree of indeterminacy disappears. Intuitively, both the mean of inflation as well as how it covaries with the stochastic discount factor across two successive periods is now pinned down by arbitrage. Thus, the indeterminacy of inflation across states of the world can be solved as long as the nominal interest rate varies with those states of the world. However, while these interest rates are varying over states, over time they are still pegged in the sense of the interest rate peg. Thus, the problem of controlling P_0 remains.

Similar steps show that if the central bank announces both its current interest rates on reserves, as well as its expected value for tomorrow, this again provides two equations with which to solve for inflation across states.

Historically, the source of going long has often been the Treasury rather than the central bank. Especially in the aftermath of wars, when long-term government debt was high, the Treasury imposed low long-term interest rates. Whether it is the Treasury or the central bank that uses its power to steer interest rates, the economics underlying the effect on inflation is the same.

3.4 Escape clauses can provide a terminal condition in finite time

Returning to feedback rules for interest rates, some research has provided terminal conditions based on escape clauses. The idea of an escape clause is that the central bank commits to a feedback rule only while inflation does not go on an explosive path. If inflation exceeds a pre-announced threshold, the central bank would switch to a different policy approach. Realistically, if inflation was rising without bound, no central bank would stick to following blindly a Taylor rule that tells it to raise interest rates more and more, even as it sees inflation rising faster and faster.

3.4.1 On-equilibrium policy switches

If the approach dictated by the escape clause pins down the price level at the date of the switch, then it provides the terminal condition for the feedback rule.

Formally, the central bank follows the feedback rule only while inflation is within some interval $\left[\pi^L, \pi^H\right]$. If, at some date T, inflation π_T falls outside this interval, then it switches to a different policy at $T + 1$. Take as given that this other policy is able to determine uniquely π_{T+1} as close as possible to the target π_{T+1}^*. It could, for instance, set a real payment on reserves as we already saw, or involve a different approach like fixing the supply of banknotes. This paper will discuss many approaches to pin down π_{T+1} further on.[14]

Going back to the solution for inflation with a Taylor rule, by iterating the Fisher equation up until a finite date T, we reach:

$$
\pi_t = \pi_t^* + \sum_{j=0}^{T-t} \phi^{-j-1} \mathbb{E}_t \left[r_{t+j} + \pi_{t+1+j}^* - \phi \pi_{t+j}^* - x_{t+j} \right] + (1+\phi)^{-T+t} \mathbb{E}_t \left(\pi_{T+1} - \pi_{T+1}^* \right).
$$
(18)

If the last term on the right-hand side is uniquely pinned down by the switch in regime, then inflation on the left-hand side is uniquely pinned down as well. If the switch leads to an inflation close to target, then the last term will be close to zero. Therefore, the effectiveness is still approximately given by the formula for ε_t that we derived earlier for the Taylor rule.

Of course, if either the width of the interval $\left[\pi^L, \pi^H\right]$ goes to zero, or the errors ε_t are large enough, then T would be close to 0. The economy would switch policy right away. In that case, one might as well dismiss Taylor rules entirely and focus on those alternatives from the start. The case for the Taylor rule must then be that it achieves lower ε_t than the alternatives, that T is expected to be large, and so that leaving the interval is infrequent.

3.4.2 Off-equilibrium threats

Regime switches can be used differently, not as terminal conditions, but as off-equilibrium threats that ensure that the regime switch never happens.[15]

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[14]The classic analysis is Obstfeld and Rogoff (1983), and see also Taylor (1996) and Christiano and Rostagno (2001).

[15]Much of this work builds on Bassetto (2005), and includes Atkeson, Chari and Kehoe (2010)'s sophisticated equilibria, Christiano and Takahashi (2018)'s strategy equilibria and Loisel (2019)'s implementability criteria.

Say policy is still committed to a Taylor rule while inflation stays in a bounded interval $[\pi^L, \pi^H]$. If at date T inflation π_T is outside of it, there is a switch in policy at $T+1$. The new policy is able to uniquely pin down inflation π_{T+1} as before.

The difference is that now this exit will never happen because it is inconsistent with equilibrium. The new policy is designed to pin down inflation to some level well inside the interval, and in particular to a level such that $\pi_{T+1} < \pi^H - r_t$.[16] The Fisher equation (9) at date T together with the regime switch pins down $i_T = \pi_{T+1} + r_t < \pi^H$. At the same time, the Taylor rule at T implies that since π_T was larger than π^H, and given that the Taylor rule coefficient is larger than one, $i_T > \pi^H$. This is a contradiction.

The only way to avoid the contradiction is for inflation to never leave the bounded interval. If the width of the interval is large enough such that the size of the exogenous shocks would never send the economy outside the interval, then the explosions that led to indeterminacy with a Taylor rule are ruled out but needed fluctuations due to changes in the inflation target are not. As the feedback rule implies that inflation explodes at rate ϕ^{-1}, then one of the bounds will be reached for sure in finite time for any inflation path that does not satisfy the elusive terminal condition. Thus the condition holds.

Just like in the previous case, the central bank is making the promise that it will not stick to the Taylor rule if inflation enters one of the explosive paths that violate the terminal condition. But now, the escape clause is inconsistent with equilibrium, and so it is assumed that rational agents would never expect it to be used. This requires a great deal of commitment by the central bank.

3.5 Non-rational expectations as an alternative to terminal conditions

Controlling inflation with a Taylor rule requires that people do not start expecting that inflation in an arbitrary far away future will grow (or fall) at an explosive rate. The explosive inflation path leans heavily on rational expectations regarding the interaction between the policy rule and equilibrium into the infinite future. Therefore, assumptions on how expectations of these far away events are formed can deliver determinacy.

Macroeconomic models with non-rational expectations involve two related concepts.[17] The first is a *temporary equilibrium*: a competitive equilibrium at each point in time defined as a function of exogenously-given expectations as well as the model's fundamentals. To

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[16]This may require a model where r_t is endogenous, so that the policy switch can credibly deliver this outcome.

[17]See Woodford (2013) for a review.

illustrate how it works, we focus on the case where the central bank follows a Taylor rule and the two fundamentals, r_t and x_t^i, are constant. In this case, the non-explosive rational expectations equilibrium according to equation (10) is a constant inflation rate: $\pi^{RE} = (r - x^i)/(\phi - 1)$.

To derive a temporary equilibrium, combine the Fisher equation (9) with the Taylor rule in equation (8) to get $r + \mathbb{E}_t(\pi_{t+1}) = x^i + \phi \pi_t$. At an arbitrary future date $t + s$, taking the public's expectations as of t, and multiplying by β^s, this is equal to:

$$\beta^s r = \beta^s \mathbb{E}_t \left(x^i + \phi \pi_{t+s} - \pi_{t+s+1} \right). \tag{19}$$

Adding these for all the s delivers:

$$\frac{r - x^i}{1 - \beta} - \phi \pi_t = (\beta \phi - 1) \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t \left(\pi_{t+s+1} \right) \equiv e_t, \tag{20}$$

The sum on the right-hand side of the equation depends on agents' expectations of future inflation. It is the sufficient statistic for how expectations affect outcomes this period, and we denote it by e_t. This defines the temporary equilibrium representation:

$$\pi_t = \frac{1}{\phi} \left(\frac{r - x^i}{1 - \beta} - e_t \right), \tag{21}$$

mapping expectations e_t to the outcome π_t.

The second pillar of a model that does not assume rational expectations is a specification of how expectations evolve given model outcomes. There are two leading models of this in the literature on inflation: one postulates forecasting rules that map past outcomes to current expectations (which is sometimes called learning); the other models how agents reach their expectations by iterating in their heads the map between initial expectations and the expectations that are consistent with the model outcomes (a process sometimes called eduction).

3.5.1 Learning

Learning models assume that expectations are formed by agents that behave like statisticians using past data to form their beliefs. Learning gives a mapping from past outcomes to current expectations. Combining it with a temporary equilibrium linking expectations to outcomes delivers the equilibrium. Expectations adjust over time, as outcomes change

expectations that in turn lead to new outcomes.[18]

To see this at work, in our simple model of the Taylor rule and constant real interest rates, agents-statisticians will hypothesize that inflation is a constant. At date t, this expectation is π_t^e, so the sufficient statistic defined in equation (20) becomes: $e_t = (\frac{\beta\phi-1}{1-\beta})\pi^e$. The temporary equilibrium in equation (21) is now equal to:

$$\pi_t = \frac{r-x^i}{\phi(1-\beta)} - \frac{\beta\phi-1}{\phi(1-\beta)}\pi_t^e. \tag{22}$$

Among different models of learning, the most popular is least-squares learning, where agents use least-squares regressions on past outcomes to form their beliefs. Since the sample mean is the least-squares estimator of a constant, then $\pi_t^e = (1/t)\sum_{s=1}^{t}\pi_{t-s}$. At time $t+1$, the new expectations of inflation π_{t+1}^e will incorporate this realized inflation rate π_t, and in turn this leads to a revised least-squares expectation. Taking the limit of this process as $t\to\infty$ delivers what is known as the *learnable* equilibrium. Tedious algebra shows that this is the non-explosive rational expectations equilibrium.

The literature often resorts to a result, the *e-stability* principle, that establishes that learning converges to the non-explosive rational expectations equilibrium if certain stability conditions hold. To obtain such conditions, start from the agents' perceived law of motion for inflation $\pi_t = \pi_t^e$, and combine it with the actual law of motion implied by the model structure, equation (22). E-stability is represented by a dynamic process in which parameters move gradually to close the gap between the actual and the perceived law of motion:

$$\frac{\partial\pi_t^e}{\partial t} = \underbrace{\frac{r-x^i}{\phi(1-\beta)} - \frac{\beta\phi-1}{\phi(1-\beta))}\pi_t^e}_{\pi_t} - \pi_t^e. \tag{23}$$

The e-stable condition is the one that ensures there is a stable solution to this dynamic equation. Simple algebra shows that π_t^e converges uniquely to π^{RE} as long as the Taylor principle, $\phi > 1$, holds.[19]

3.5.2 Eduction

There is a variety of models of eduction that we cannot cover here. The central idea is that agents go through a mental process whereby they iterate on what expectations to have,

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[18]Classics in the literature are Evans and Honkapohja (2001), Bullard and Mitra (2002), McCallum (2003).
[19]McCallum (2007) discusses the link between e-stability and determinacy with a terminal condition.

and what their implications are for equilibrium inflation, until the two converge. This convergence need not happen at the fixed point of rational expectations, nor does it have to happen over time, like with learning, but rather occurs in their mind.

An example of eduction is reflective expectations.[20] Let $e_t(k)$ be the expectations after k stages of reflection. This model assumes that these expectations are updated to close the gap to the model-consistent expectations e_t^*. A continuous specification of this process over reflexion round k is:

$$\frac{\partial e_t(k)}{\partial k} = e_t^*(k) - e_t(k). \tag{24}$$

Combining the definition of e_t in equation (20), with the temporary equilibrium representation in equation (21), gives:

$$e_t^* = \frac{\beta\phi - 1}{\phi} \frac{r - x^i}{(1 - \beta)^2} - \frac{\beta\phi - 1}{\phi} \sum_{s=0}^{\infty} \beta^s e_{t+s+1}. \tag{25}$$

This is the map from conjectured expectations e_{t+s+1} to model-consistent expectations e_t^*. Model-consistent expectations are those that would be consistent with the model if people acted under the initially conjectured expectations e_t.

Rational expectations are the fixed point of this mapping, when $e_t = e_t^*$. It is easy to see that in the non-explosive case, the rational expectations of this sufficient statistic are: $e^{RE} = \left(\frac{\beta\phi - 1}{1 - \beta}\right) \pi^{RE}$. With non-rational expectations, depending on the rounds of inner reflection that agents go through, k, then the outcome will be different. The question this literature poses is whether, in the limit as $k \to \infty$, only one of the rational expectations equilibrium is selected, namely the non-explosive one.

Answering this question requires a few more assumptions on the initial expectations that agents start with, $e_t(0)$. Following García-Schmidt and Woodford (2019), assume that in terms of their dynamics over time, expectations converge exponentially at a fixed rate $\lambda < 1$ so: $e_{t+s+1}(0) = e_\infty(0) + \lambda^s a(0)$, $\forall s \geq 0$. This reduces the problem of finding the sequence of expectations to finding three finite scalars: $e_0(k)$, $a(k)$ and $e_\infty(k)$. The process

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[20]See García-Schmidt and Woodford (2019), building on the calculation equilibrium of Evans and Ramey (1992)

of updating these three scalars following equation (24) follows the dynamic system:

$$\frac{\partial e_t(k)}{\partial k} = \underbrace{\begin{pmatrix} -1 & \frac{1-\beta\phi}{\phi}\frac{1}{1-\beta\lambda} & \frac{1-\beta\phi}{\phi}\frac{1}{1-\beta} \\ 0 & \frac{\lambda-\phi}{\phi(1-\beta\lambda)} & 0 \\ 0 & 0 & \frac{1-\phi}{\phi(1-\beta)} \end{pmatrix}}_{\equiv A} \begin{pmatrix} e_0(k) \\ a(k) \\ e_\infty(k) \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{\equiv B} \frac{\phi-1}{\phi}\frac{r-x^i}{(1-\beta)^2}, \qquad (26)$$

Since the A matrix is triangular, its eigenvalues are given by its diagonal elements. It is clear that as long as the Taylor principle holds, $\phi > 1$, then all the eigenvalues are negative. Therefore, as $k \to \infty$, expectations converge to the unique point $-(A^{-1}B)\frac{\phi-1}{\phi}\frac{r-x^i}{(1-\beta)^2}$. More algebra shows that this point is the non-explosive rational expectation belief e^{RE}.

To conclude, in the limit where agents engage in infinite reflection, the outcomes where inflation explodes to infinity are no longer equilibria. Non-rational expectations delivers determinacy of inflation with a Taylor rule, as an alternative to the terminal condition.[21] Other non-rational expectations models based on eduction have been shown to achieve the same result.[22]

3.6 Global analysis, banknotes, and the zero lower bound

Unlike feedback rules, both the payment on reserves and the going long approaches do not require log-linearizing the Fisher equation. We now re-examine the lessons from the Taylor rule without log-linearization.

3.6.1 Global analysis of the Taylor rule

To simplify, we consider the case where there is no uncertainty, so the stochastic discount factor is a constant β and the inflation target Π_t^* is deterministic. Therefore, there is possibly indeterminacy only with respect to the initial price level. The Taylor rule combined with the Fisher equation is then:

$$\frac{\Pi_{t+1}}{\beta} = \Pi_t^\phi \underbrace{\left(\frac{\Pi_{t+1}^*}{\beta\Pi_t^{*\phi}} \right)}_{X_t^{i*}}. \qquad (27)$$

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[21]A valid criticism is that the assumption that over time the initial expectations converge exponentially to a finite $e_\infty(0)$ goes some of the way towards excluding the explosions of inflation, but by assumption.
[22]Examples are k-level thinking (Farhi and Werning, 2019) or lack of common knowledge (Barrdear, 2018).

The left-hand side is the nominal interest rate given by the Fisher equation. The right-hand side is the non-linear Taylor rule with $\phi > 1$ and X_t^{i*} chosen to be most effective with no uncertainty. Rearranging, this simplifies to $\Pi_{t+1}/\Pi_{t+1}^* = (\Pi_t/\Pi_t^*)^\phi$, a nonlinear difference equation.

Taking logs gives precisely the same dynamics as in the log-linearized case. If inflation starts on target, it stays there forever. If it deviates upwards or downwards, then this leads to inflation exploding to plus or minus infinity at the rate $\ln(\phi)$. Inflation control depends again on a terminal condition that rules out these equilibria.

### 3.6.2   Banknotes and the ELB reduce effectiveness

This global analysis allows us to address the issue of negative deviations of inflation from target. At some point, as inflation explodes downwards, it goes below the real interest rate so the nominal interest rate must be negative as well. Yet, central banks typically issue physical banknotes, together with reserves. Since both are supposed to serve as the unit of account, they exchange one-to-one. Banknotes have the property that they pay no interest, and given storage costs and risk of theft, they have some gross nominal return of $\xi < 1$. This puts a constraint on the interest on reserves, namely that banks would want to substitute all of their reserves for banknotes if interest rates went below ξ. Banknotes therefore imply an effective lower bound (ELB) on what the payment of interest on reserves can be.

Any monetary policy rule that promises to choose the remuneration of reserves in a way that is inconsistent with the ELB is not admissible. With the payment on reserves rule (or the Wicksellian rule) this lower bound makes no difference to determinacy but an important one to effectiveness. The payment on reserves rule has to be modified to: $1 + I_{i,t+1}^v = \max\{(1 + X_t^i)P_{t+1}, \xi\}$. When the ELB does not bind, say at date T, one gets the same equations as before, pinning down the price level on target at P_T^*. When it does bind, then the Fisher equation implies that $P_{t+1} = \beta\xi P_t$. Because P_T was pinned down, so are all the prices before then, and the price level is still determinate. However, now, $P_t = (\beta\xi)^{t-T}P_T^* \neq P_t^*$. During the ELB periods, the economy is in a deflation, no matter what is the inflation target, and the central bank can do nothing about it.

3.6.3   The peril of Taylor rules with the ELB

With the Taylor rule, the problem gets more serious. From the Fisher equation, the ELB constraint $1 + I_t \geq \xi$, implies that $P_{t+1}/P_t \geq \beta\xi$. Combining with the Taylor rule, the

Figure 1: Inflation dynamics at the ELB with a Taylor rule
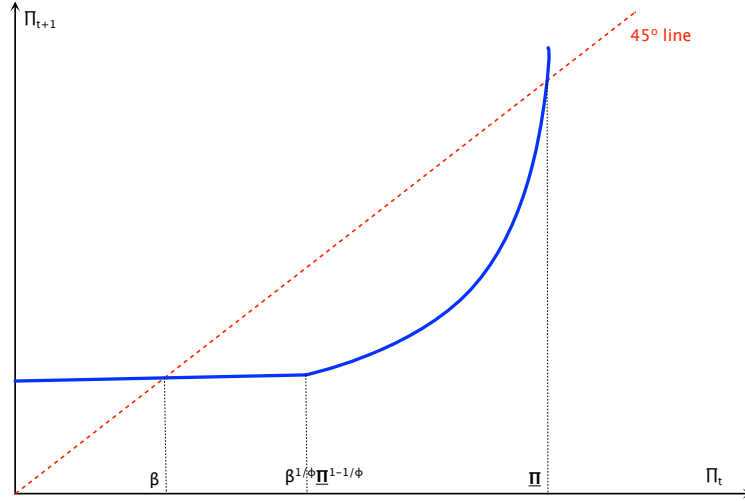
difference equation for inflation becomes:

$$1 = \frac{\beta}{\Pi_{t+1}} \max \left\{ \frac{\Pi_{t+1}^*}{\beta \xi} \left(\frac{\Pi_t}{\Pi_t^*} \right)^{\phi}, 1 \right\}. \tag{28}$$

This difference equation for inflation is represented in figure 1.[23] As soon as inflation is equal to $\beta \xi$, it stays there forever. This is a global steady state equilibrium of the system: a deflation trap. Moreover, note that if P_0 is below target, the system will converge to the deflation trap. Since any such deviation leads to this same outcome, consistent with the equilibrium conditions, then the price level is again indeterminate: any initial inflation between Π_0^* and $\beta \xi$ is consistent with an equilibrium, and is not ruled out by excluding explosive solutions.[24]

Setting aside indeterminacy, the presence of two steady states in this system, one where $\Pi_t = \Pi_t^*$ and another where $\Pi_t = \beta \xi$ implies that in a system with shocks to either the state of the economy, the inflation target, or policy mistakes, there will be two stochastic solutions fluctuating around these steady states. Then, if there is a sunspot that triggers a change between them, equilibrium inflation will alternate between being close

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[23]The classic paper in this analysis is Benhabib, Schmitt-Grohe and Uribe (2001).

[24]However, Christiano, Eichenbaum and Johannsen (2018) show that an e-stability restriction on the set of equilibria delivers uniqueness.

to target or being close to the deflation trap.[25] Depending on the exogenous distribution of this sunspot, the effectiveness of policy can be arbitrarily poor.

There are different ways to rule out the deflation equilibrium. The first, and easiest, is to not use the Taylor rule. Similarly, one can switch from the Taylor rule to any of the alternative approaches as an escape clause whenever the economy threatens to converge to deflation.

A second, and harder, solution is to eliminate or relax the ELB constraint by lowering ζ, perhaps all the way to zero. Some suggestions in the literature on how to lower ζ are to eliminate banknotes, charge a tax on them, or default on the commitment to exchange currency and reserves one-for-one.[26]

A third solution is to avoid remuneration of reserves as the policy tool. Since this problem arises because of the presence of banknotes, the next section focuses on the supply of banknotes directly as an alternative tool to control inflation.

4   The monetarist approach: currency and seignorage

Banknotes, or currency, are distinct from reserves in five ways. First, they can be freely held by anyone in the economy, not just banks. Second, they are physical and the central bank can produce them at close-to-zero cost. Third, they are anonymous as people do not have to declare to the government how much currency they have or from whom they got it. Fourth, for some payments it may be easier to use banknotes than electronic means backed by reserves (and for others the opposite). Fifth, banknotes pay no interest.

The first four properties create a demand for the services provided by banknotes separate to the demand for reserves. Some economic agents prefer to not use the banking system when making payments, some prefer physical to digital payments, some want anonymity in their transactions, and some find cash easier to use. The fifth property implies that the opportunity cost of using banknotes as opposed to the digital means of payment provided by the banking sector is the interest rate paid on reserves. Using $H_t \geq 0$ to denote banknotes held by the public, then the demand for real currency balances H_t/P_t depends negatively on the opportunity cost I_t.

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[25]See Mertens and Ravn (2014) or Borağan Aruoba, Cuba-Borda and Schorfheide (2017).
[26]See Goodfriend (2016); Rogoff (2017); Agarwal and Kimball (2019) for extensive discussions.

An iso-elastic form of this demand function is:

$$\frac{H_t}{C_t P_t} = e^{u_t^d} \left(\frac{I_t}{1 + I_t} \right)^{-\tilde{\eta}}, \tag{29}$$

where u_t^d represents a shock to the demand for currency. The income elasticity of the demand for banknotes is set to one to be consistent with the (rough) balanced growth fact that the left hand-side of this equation (sometimes called the inverse of velocity) does not have a strong trend over decades in the data. The interest rate elasticity is measured by the constant $\tilde{\eta} > 0$.

There is a long empirical literature devoted to estimating this function.[27] A common difficulty is that there are large shifts across years as a result of changes in the availability of ATMs, in the social norms of what shopkeepers will accept as payment, and in the prevalence of crime that drives the demand for the anonymity of banknotes. These all change the relative ease of using banknotes versus alternatives. The fit of models of currency demand tends to be poor, and the estimated variance of u_t^d is large.

Turning to the supply of banknotes, the central bank can in principle perfectly control H_t^S. However, because of the existence of close substitutes to currency produced by the private market, there is a disconnect between the banknotes the central bank prints, and the money that people find useful. A large old literature devised different measures of monetary aggregates to deal with this. We allow for it simply by writing the market clearing condition as

$$H_t = e^{u_t^s} H_t^S, \tag{30}$$

where u_t^s is very volatile, and exogenous with respect to the price level.

Combining these two equations, for currency supply and demand, together with the Fisher equation and using the log-linearized version such that $h_t = \log(H_t^S / \bar{H})$ and $\eta = \tilde{\eta}/(1 + I)$, we get the key equation of the monetarist approach:

$$h_t - p_t = c_t - \eta(r_t + \mathbb{E}_t \, \pi_{t+1}) + u_t^d - u_t^s. \tag{31}$$

The economic force that drives the price level behind this equation works as follows.

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[27]Recently, see Benati et al. (2019), who estimate equation (29) on data for any countries to obtain a $\tilde{\eta}$ between 0.3 and 0.6, or a more general micro-founded model by Alvarez and Lippi (2014) whose time-series behavior is approximately captured by equation (29) and gives $\tilde{\eta}$ between 0.25 and 0.46. Ireland (2009) and Ball (2001) argue that a demand system relating log real currency balances to the level of interest rates fits the data better, but the former estimates a semi-elasticity of demand of 1.8-1.9, while the latter estimates it to be only 0.05 once he allows the income elasticity to be below 1 (and estimated to be 0.5).

All else equal, a higher price level today lowers real currency balances supplied by the central bank. At the same time, it lowers expected inflation between the present and the next period, which lowers the nominal interest rate and raises the demand for banknotes. With lower supply and higher demand for banknotes, the price level must fall. This re-equilibrates the market by both increasing the supply, and by lowering demand through a higher nominal interest rate.

The logic is soothingly familiar because it reintroduces Marshallian partial-equilibrium supply and demand to think about the price level in terms of the service provided by banknotes. At the same time, it can be misleading because p_t is not the price of the banknotes. Changes in p_t bring the market to equilibrium by affecting the actual cost of currency i_t and by directly changing the quantity of real currency that is held.

4.1 Micro-foundations and the terminal condition

The currency demand equation in (29) can be derived within the simple general-equilibrium micro-foundations laid out in section 2.5. Simply change the utility function to:

$$u(C_t, H_t/P_t) = \left[C_t^{1-1/\tilde{\eta}} + e^{u_t^d/\tilde{\eta}} (H_t/P_t)^{1-1/\tilde{\eta}} \right]^{\tilde{\eta}/(\tilde{\eta}-1)}. \tag{32}$$

and the budget constraint to: $P_t C_t + B_t + H_t \leq P_t Y_t + B_{t-1}(1 + I_{t-1}) + H_{t-1}$. Equation (29) follows from the first-order conditions for an optimum.

More interesting is another first-order condition, the transversality condition. It states that at infinity the utility value of assets held by the consumer must be zero, for otherwise she would be better off consuming more and saving less. In this problem, since the only asset in non-zero net supply is currency, the transversality condition reduces to:

$$\lim_{T \to \infty} M_{t,T} \left(\frac{H_T}{P_T} \right) = 0. \tag{33}$$

This provides a terminal condition that will prove to be crucial in delivering determinacy. Unlike in the previous section, this one follows directly from optimal behavior.

Moreover, note that we can rewrite the equilibrium in the currency market in equation (31) as:

$$i_t = \frac{p_t}{\eta} + \frac{c_t + u_t^d - u_t^s - h_t}{\eta}. \tag{34}$$

This is mathematically equivalent to a Wicksellian interest rate feedback rule. Since $1/\eta >$

0, it satisfies the determinacy condition. But while in section 3 this was a policy rule, here it emerges as an equilibrium condition. It should be no surprise that as long as $\eta > 0$, inflation will be determinate if h_t is exogenous with respect to p_t. We show this next.

4.2 Money growth rules

The classical monetarist rule proposes that the supply of currency grows at a constant rate over time. That is $h_t = \bar{x}^h t$, where $\bar{x}^h$ is a constant. Replacing into equation (31) gives a difference equation for the price level:

$$(1 + \eta)(p_t - \bar{x}^h t) = \eta \, \mathbb{E}_t(p_{t+1}) - \eta \bar{x}^h t + \eta r_t - c_t - u_t^d + u_t^s. \tag{35}$$

As usual, we can iterate this forward. The transversality condition in equation (33) ensure that the limit term is zero since the nominal interest rate converges to a finite value. The price level is thus determinate and given by:[28]

$$p_t = \bar{x}^h t + \eta \bar{x}^h + \frac{1}{1+\eta} \sum_{j=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^j \mathbb{E}_t[\eta r_{t+j} - c_{t+j} - u_{t+j}^d + u_{t+j}^s]. \tag{36}$$

If one dismisses currency demand and supply shocks, then in a long-run balance growth path where consumption grows at a constant rate, inflation is simply equal to the money growth rate $\bar{x}^h$ minus the growth rate of consumption. Thus, choosing $\bar{x}^h$ to be the long-run inflation target of the central bank plus the real growth rate of the economy provides an effective way to achieve the target. However, with financial innovation, the central bank cannot dismiss the shocks to the demand and supply of currency.

The focus of the central bank then becomes measuring trends in u_t^d and u_t^s, so as to choose $\bar{x}^h$ in order to deliver the inflation target. Every year though, consumption growth varies with the business cycle, and there are large shocks to the supply and demand for currency. The terms in the sum above move around significantly, and so will inflation.

The most effective rule for currency supply chooses: $h_t = p_t^* + \hat{c}_t - \eta(\hat{r}_t + \hat{p}_{t+1}^* - p_t^*) + \hat{u}_t^d - \hat{u}_t^s$, so as to accommodate the business cycle as well as anticipated shifts in the

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[28]Different micro-foundations for money imply different forms of the money demand function, and as such potentially slightly different conditions for determinacy of the price level. Still, the basic result and economic intuition in this paragraph remains, see Carlstrom and Fuerst (2003).

demand and supply for currency. The effectiveness of this policy is given by

$$\varepsilon_t = \frac{1}{1+\eta} \sum_{j=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^j \mathbb{E}_t[\hat{c}_{t+j} - c_{t+j} - \eta(\hat{r}_{t+j} - r_{t+j}) + \eta(p^*_{t+1+j} - \hat{p}^*_{t+j+1})$$

$$+ (\hat{u}^d_{t+j} - u^d_{t+j}) - (\hat{u}^s_{t+j} - u^j_{t+j})]. \tag{37}$$

The deviations of inflation from target can be quite large. In fact when it was tried in the UK in the early 1980s, this monetary policy rule led to volatile nominal interest rates associated with volatile expected annual inflation rates. Compared to nominal interest rate rules, monetarism seems to lead to more volatile short-term inflation.

The link to interest rates under a monetarist approach for monetary policy arises because the nominal interest rate i_t is now endogenous so that the market for currency clears. Canzoneri, Henderson and Rogoff (1983) blur this distinction by specifying a feedback rule for currency that depends on the nominal interest rate: $h_t = x_t^h + \phi i_t$. In this case, the central bank can limit the volatility of the nominal interest rate. In fact it can even peg it to follow a pre-determined path, while inflation remains determinate.

4.3   Seignorage rules

When the central bank prints currency, it could buy goods with it giving rise to a resource flow called seignorage. Since it costs close to nothing to produce currency and there is a downward-sloping demand for it, currency is not a liability of the central bank, but rather a durable good that it produces and sells for its value $1/P_t$. Seignorage is then equal to:

$$S_t^H = \frac{H_t^S - H_{t-1}^S}{P_t} = C_t \left[e^{u_t^d - u_t^s} \left(\frac{I_t}{1+I_t}\right)^{-\tilde{\eta}} - e^{u_{t-1}^d - u_{t-1}^s} \left(\frac{I_{t-1}}{1+I_{t-1}}\right)^{-\tilde{\eta}} \frac{P_{t-1}C_{t-1}}{P_t C_t} \right]. \tag{38}$$

The second equality used the expression for currency demand that we derived earlier.

This expression makes clear that seignorage and inflation are tightly linked. Higher expected inflation comes with higher nominal interest rates, which lowers the demand for currency and lowers seignorage. At the same time, a higher unexpected inflation implies that more goods can be bought with the newly printed banknotes, which raises seignorage.

Consider then a policy rule for seignorage. The central bank that follows it is committed to generating some revenues, just like a government fiscal agency that has a target

for tax revenues, or a State-owned company providing a public service with a target for profits. Historically, this was common, as central banks have for centuries been asked to provide fiscal resources for the sovereign. Only during the past few decades did inflation targeting replace seignorage as a primary task for the central bank.

Given an exogenous target for seignorage s_t^h, the central bank prints more or fewer banknotes as needed to reach this target. Log-linearizing the relation between seignorage and inflation in (38) provides a second-order difference equation for the price level. Given an initial p_{-1} and the transversality condition, this equation determines inflation.

This is not a theoretical curiosity, but a policy that some central banks have frequently adopted as they followed instructions from the Treasury. Moreover, fiscal reforms that lowered the government's demand for fiscal revenue from the central bank (large falls in s) were historically associated with the end of hyper-inflations.[29]

At the same time, the effectiveness of this approach is poor. In annual data, the large shocks to u_t^d and u_t^s lead to volatile inflation. In the long run, this approach has often led to hyper-inflation. The reason is that in steady state, equation (38) implies that $S \leq Ce^{u^d - u^s}$. If the central bank aims to raise revenue beyond this limit, then inflation is again indeterminate. Furthermore, this upper bound—the peak of the Laffer curve for the inflation tax—is hard to estimate, moves around, and small changes in S close to its peak come with large changes in inflation. [30] Turning the central bank into a fiscal agent often leads to run-away inflation.

4.4 Non-satiated market for reserves

No modern central bank sets H_t^S exogenously. Rather, central banks stand ready to exchange reserves for banknotes one-to-one at all times, so they control only the sum $V_t + H_t^S$, the monetary base. People can freely choose to substitute between the two components.

In the model described so far, the quantity of V_t was irrelevant for both the real equilibrium as well as for inflation. While currency provided some service, reserves did not. This is a good description of the situation in the major western economies after the first rounds of quantitative easing that followed the financial crisis. The demand for reserves was satiated by increasing V_t, and their market was saturated, so that further changes in

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[29]See Sargent (1982) and Sargent and Wallace (1984).
[30]Locally, by definition of a maximum, $\partial \pi / \partial x = \infty$.

the quantity of reserves had no discernible effect on inflation.[31]

Maybe this will not be so in the future, and reserves will be scarce so that there is a non-horizontal demand for them. In that case, the determination of inflation will depend on a hybrid of the monetarist and the no-arbitrage approaches.

4.4.1 Reserves as money

Start with the case where there is a downward-sloping demand for reserves, just as there was one for currency.[32] Perhaps this will arise because central banks offer digital deposits to households, and not just banks, so that the benefits from using currency for payments will extend to reserves as well.

The opportunity cost of holding reserves is the gap between the return from holding financial assets instead of reserves, so it is given by $i_t - i_t^v$. We can write the demand curve for reserves as:

$$v_t - p_t = c_t - \eta_v(i_t - i_t^v). \tag{39}$$

Now, the central bank can choose both v_t and i_t^v. In particular, consider the case where it follows a Wicksellian rule, whereby the interest on reserves responds to p_t with a coefficient ϕ. In that case, the price level is determinate as long as $\phi > -1/\eta_v$. This includes the case where $\phi = 0$, that is where there is a pure interest rate peg. The logic is that of the monetarist approach. With two policy tools, the central bank can potentially get closer to tracking the variables it must offset to keep inflation close to its target.

4.4.2 Bank deposits as money

Most households use their bank deposits to engage in transactions. A similar demand equation would hold but with respect to h_t^d, bank deposits, and their opportunity cost is the gap $i_t - i_t^d$ where the interest rate paid on deposits is i_t^d.

The central bank does not control either h_t^d or i_t^d in this case, since both are determined by the equilibrium in the banking sector. However, banks also deposits reserves at the central bank and could invest in financial assets. Optimality in their portfolio choice leads to a log-linearized relation of the form $i_t - i_t^d = \ell(i_t - i_t^v)$.[33] In section 3, competitive frictionless banks in equilibrium implied $\ell = 1$, leading to $i_t = i_t^v$. With market power of

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[31]Reis (2016) shows evidence for satiation of reserves in the US and Reis (2019b) argues this is desirable.
[32]See Diba and Loisel (2019).
[33]See Piazzesi, Rogers and Schneider (2019).

banks, or financial frictions, $\ell < 1$. Combining these two equations:

$$h_t^d - p_t = c_t - \eta_d \ell(i_t - i_t^v). \tag{40}$$

If the production of deposits by banks was exogenous with respect to the price level, then by choosing the interest on reserves, the central bank can again control inflation. Even though the policy tool is the interest rate, the economic logic is the monetarist one, as the key equation is the demand curve above, and the terminal condition comes from transversality. If, instead, the quantity of reserves affects the amount of deposits—a money-multiplier process—then we are back at the previous case where the central bank has two tools, v_t and i_t^v, with which to improve the effectiveness of inflation control.

Either way, while central banks' digital currencies, more realistic banking sectors, or scarce reserves all affect the dynamics of inflation, the economic logic and the policy approach by which the central bank can control it are unchanged.

5   The solvency approach: fiscal theory of the price level

The characterization of central banks' activities has so far considered an expanding list of items in its balance sheet such as reserves, currency, and possibly long-term bonds. In general, the central bank can hold a portfolio of assets with current value A_t, that may be public or private, and that have a risky return $(1 + R_{t+1}^a)$ next period. The central bank also has real expenses with its staff E_t, and pays dividends to the fiscal authority D_t. Its flow of funds is then given by:

$$V_t + H_t - A_t P_t = (1 + I_{t-1}^v)V_{t-1} + H_{t-1} - P_t(1 + R_t^a)A_{t-1} + P_t(E_t + D_t). \tag{41}$$

Let the net surplus of the central bank after paying dividends be: $S_t \equiv S_t^H + (1 + R_t^a - (1 + I_{t-1}^v)P_{t-1}/P_t)A_{t-1} - E_t - D_t$ (where S_t^H was defined in equation (38)). The central bank earns income from seignorage and from the return on its assets in excess of what it pays on the reserves. It incurs in spending on its staff as well as in dividends paid to the government. The flow budget constraint of the central bank can then be written more compactly as:

$$\frac{V_t}{P_t} - A_t + S_t = \frac{(1 + I_{t-1})V_{t-1}}{P_t} - \left(\frac{(1 + I_{t-1})P_{t-1}}{P_t}\right)A_{t-1}. \tag{42}$$

This accounting identity becomes a meaningful restriction when it is combined with the economic assumption that the central bank cannot be running a Ponzi scheme on its reserves. Thus reserves cannot exceed the value of assets plus the present value of expected surpluses of the central bank. This intertemporal constraint is:

$$\frac{(1 + I_{t-1})(V_{t-1} - P_{t-1}A_{t-1})}{P_t} = \mathbb{E}_t \left(\sum_{j=0}^{\infty} M_{t,t+j} S_{t+j} \right). \tag{43}$$

So, net debts (or savings) of the central bank must equal the present value of its expected surpluses (or deficits). Operationally, this points to a new tool that the central bank can use to control inflation: the size and composition of its balance sheet.

5.1 The fiscal theory of the price level

The determination of inflation follows almost immediately from turning the intertemporal budget constraint of the central bank on its head. Make the simplifying assumptions that S_t is an exogenous i.i.d. process over time with mean $\bar{S}$, and that the interest on reserves I_t^v is an exogenous process X_t^i. Then, equation (43) becomes:

$$P_t = \frac{(1 + I_{t-1})(V_{t-1} - P_{t-1}A_{t-1}))}{\sum_{j=0}^{\infty} \mathbb{E}_t(M_{t,t+j} S_{t+j})} = \frac{(1 + X_{t-1}^i)(V_{t-1} - P_{t-1}A_{t-1})}{S_t + \bar{S}(\sum_{j=1}^{\infty}(1 + R_{t,t+j})^{-1})}. \tag{44}$$

The sequence of real interest rates is exogenous with respect to the price level, and V_{t-1} was set in $t-1$, so it is exogenous with respect to period t realizations. Therefore, the right-hand side is exogenous, and this provides a unique solution for P_t.[34]

A larger current or future expected surpluses, S_t or $\bar{S}$, lead to a lower price level. By controlling its surpluses, the central bank can target inflation. It can do so by choosing the composition of the assets it holds (and so the risk in their returns), or more directly by varying its expenses, or the dividends it pays out.

Setting a peg for the nominal interest rate no longer leads to indeterminacy. The choice of X_t^i pins down expected risk-adjusted inflation as we saw in section 3.1. The budget constraint does the rest, uniquely determining the price level and hence inflation fluctuations around the target.

The economic force behind this approach to controlling inflation is the following.

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[34]The original analysis is Woodford (1994) and Sims (1994) and our approach is closer to that in Cochrane (2005) and Benigno (2019).

When the surplus of the central bank falls, fewer real resources are available to back the value of its debt, which is therefore lower. Because reserves are default-free, they have a fixed value in nominal terms, and they are the unit of account, the only way for their real value to fall is for the price level to rise. The price level adjusts as banks choose to hold more or fewer reserves in response to them becoming a Ponzi scheme. They do so until the real value of reserves is again in line with the central bank's assets and surpluses.

It is the control of the real resources earned by the central bank that gives it control over inflation. Insofar as the resources of a government body are fiscal, this mechanism is called the fiscal theory of the price level (FTPL).[35]

5.2 Central bank solvency and the Ricardian condition

An alternative way of writing the intertemporal budget constraint of the central bank is in its limit form:

$$\lim_{T \to \infty} M_{t,T} \left(\frac{V_T}{P_T} - A_T \right) = 0. \tag{45}$$

This equation is equivalent to equation (43) given the flow of funds in equation (42). It is the other side of a transversality condition for the private sector, which would not want to hold a perpetual claim on the central bank. It is a solvency condition for the central bank in the economic sense of the word: like any other economic agent, the central bank is solvent if it is not running a Ponzi scheme, that is if it satisfies its intertemporal resource constraint. The FTPL approach is a solvency approach.

Taking this solvency perspective provides an alternative way to understand the economic mechanism behind the determination of the price level, and extends the analysis to budget rules.[36] Let $\hat{V}_t = V_t/P_t - A_t$ be the net real reserves of the central bank. In accounting terms, this is the negative of the central bank's net worth. Assume now that the central bank surplus follows a feedback rule:

$$S_{t+1} = \phi \hat{V}_t + X^s_{t+1}. \tag{46}$$

If the real value of reserves rises, or the net worth falls, the central bank may pay less dividends or cut spending to raise its surplus in which case $\phi > 0$. In our previous case $\phi = 0$. Combining this rule with the flow equation (42) one period forward, gives a

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[35]For criticism of this mechanism, see Buiter (2017) and for a defense Sims (2013).
[36]The original analysis is in Leeper (1991).

difference equation for real net reserves:

$$\left[\frac{(1+I_t)P_t}{P_{t+1}}\right]\hat{V}_t = \hat{V}_{t+1} + \phi\hat{V}_t + X_{t+1}^{(s)}. \tag{47}$$

Multiply by the stochastic discount factor M_{t+1} and take expectations as of date t and this reduces to:

$$\left(1 - \phi(1+R_t)^{-1}\right)\hat{V}_t = \mathbb{E}_t(M_{t+1}\hat{V}_{t+1}) + \mathbb{E}_t(M_{t+1}X_{t+1}^s). \tag{48}$$

We can iterate the difference equation forward. If the stochastic discount factor converges to the constant β, then as long as $\phi < \beta^{-1} - 1$, the terminal condition ensures that $\hat{V}_t$ will be equal to the present value of future exogenous surpluses. Recalling that $\hat{V}_t = V_t/P_t - A_t$, by pinning down surpluses, the central bank pins down the price level.

The literature has called feedback rules that satisfy this condition *non-Ricardian policies*. A central bank that follows these policies will not raise its surpluses too strongly when reserves rise. Thus, higher reserves must lead the price level to rise, so that the real value of these reserves falls back into the equilibrium where the central bank remains solvent. In contrast, Ricardian policies are those for which ϕ is larger than the threshold so the central bank's solvency is assured by it raising its surpluses when reserves rise, no matter what the price level is.

What is special about the central bank relative to other private agents is that its liabilities are the unit of account. It can always honor the payment on reserves nominally by issuing more reserves. A private agent can also issue as many liabilities as it wants, but it will find that they are worthless if it tries to run a Ponzi scheme, so it can get no real resources in exchange for the new debt it issues. The same applies to a central bank, but since the nominal price of reserves is by definition one, their real value can only fall if the price level rises. Inflation results from the condition that the central bank must stay solvent.

Similar to before, the price level is the unique solution to the equation:

$$\frac{(1+X_{t-1}^i)V_{t-1}}{P_t} - \left(\frac{(1+X_{t-1}^i)P_{t-1}}{P_t}\right)A_{t-1} - S_t = \sum_{j=0}^{\infty}\mathbb{E}_t\left[\left(\frac{M_{t,t+1+j}}{1 - \phi\prod_{j'=0}^{j}(1+R_{t+j'})^{-1}}\right)X_{t+j}^s\right].$$
$$\tag{49}$$

The effectiveness of this policy will depend on the ability of the central bank to vary the interest rate peg X_{t-1}^i and the net income rule for X_{t+j}^s, given its forecasts of real interest rates and changes in the inflation target. Note that to the extent that the composition

of the assets is risky, fluctuations in returns will affect X_{t+j}^s and the central bank would have to adjust expenses to offset these. Insofar as it is hard to do so in real time, then a new source of error from this approach comes form the risks in the central bank's balance sheet. Error minimization can explain central banks' practice of avoiding risky returns on their choice of assets.[37]

5.3 Dividend rules and fiscal dominance

Central banks are part of the government, fiscally linked to the Treasury by the dividend process $\{D_t\}_{t=0}^\infty$. Independence is defined as having control over the dividend process, and as a result some control over the surplus S_t. The extent to which it has this control is at the heart of the fiscal approach for controlling inflation.

In one case, the central bank must by law pay out as a dividend all of its marked-to-market net income. In that case, by definition $S_t = 0$. But then, the central bank cannot control inflation using the solvency approach, because it is always solvent. This is sometimes referred to as the central bank having full fiscal support.[38]

In another case, the Treasury imposes a dividend process on the central bank. The central bank is no longer independent. If the central bank is committed to a monetarist approach, then we already saw in section 4.3 that it will respond to the Treasury's demands by printing banknotes to generate seignorage. If the central bank is instead committed to a solvency approach, then by imposing a dividends process, the Treasury will effectively be controlling the S_t process, and thus determining inflation. Either through seignorage or through the solvency of the central bank, this state of affairs is often referred to as "monetizing the fiscal deficit", as it is the monetary base, currency and reserves, that is adjusting to provide the necessary funding for the Treasury.

A third case is that considered by the bulk of the literature on the FTPL.[39] It starts by noting that the fiscal authorities also face an intertemporal budget constraint linking the value of its liabilities, call them B_t, to the present value of its primary surpluses PS_t:

$$\frac{B_{t-1}(1 + I_{t-1})}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} M_{t,t+j} PS_{t+j}. \tag{50}$$

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[37]From a different perspective, central banks often accept term deposits by banks, in which case the maturity structure of reserves will affect the dynamics of inflation as in Cochrane (2001); Reis (2017).

[38]See Hall and Reis (2015) and Del Negro and Sims (2015).

[39]See Leeper and Leith (2016) for a a survey.

A first assumption of the conventional FTPL is that the government does not default on its liabilities. Unlike reserves, government bonds are not the unit of account. Moreover, sovereign default is a frequent event.[40] Still, if in some circumstances somehow the Treasury can commit to never default, then government bonds become substitutes for reserves.

A second conventional assumption in the FTPL literature is that central bank dividends can take any value, positive or negative, not chosen or controlled by the central bank. The primary surpluses of the Treasury include the dividends from the central bank, which appeared on the central bank's surplus with a negative sign. If dividends can take any value, then the two separate intertemporal budgets in equation (43) and (50) do not pose a constraint, as their right-hand side can take on any value. Only their sum, that is the consolidated budget of the whole government, constrains policies. This constraint states that the sum of reserves and government liabilities (netting out the government bonds held as assets by the central bank) must equal the present value of the primary surpluses, without the central bank's dividends but including the net income of the central bank.

The third conventional assumption of the standard papers in the FTPL literature is that the Treasury chooses this surplus. The central bank can vary its expenses or the composition of its assets, but the Treasury can always respond to undo these policies, ultimately controlling what the primary surplus is.

Combining the three assumptions, by the same logic that allowed the central bank to control inflation, it is now the Treasury that controls inflation. The solvency of the Treasury then becomes tied to the price level, just like the central bank's did. The literature refers to this scenario as fiscal dominance of the Treasury over the central bank. Now, the control errors arise from the side of the government and its fiscal surplus.

6 The gold standard approach: choosing pegs

So far, we have focussed on approaches to determine inflation based on savings choices across private assets and reserves, currency, or other government liabilities. Next, we focus instead on consumption choices to describe one of the most classic forms of inflation control: pegging the exchange rate of the unit of account to an external good or currency.

With many non-durable goods available for consumption, the optimality condition of

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[40]Reinhart and Rogoff (2009).

the household is that the marginal rate of substitution between any two goods, i and 0, must equal their relative prices. In logs:

$$\rho_t(i) = p_t(i) - p_t(0). \tag{51}$$

The marginal rate of substitution is determined by the amounts of each good consumed and produced which are exogenously determined by the endowments, just like the stochastic discount factor earlier.

In turn, the price level is an index over the price of all goods, which we take as being a simple geometric average:

$$p_t = \sum_{i=0}^{I} \omega_i p_t(i) = p_t(0) + \sum_{i=1}^{I} \omega_i \rho_t(i), \tag{52}$$

where the second equality comes from using the optimality condition just above. The weights ω_i are non-negative and sum to one. They are independent with respect to the overall price level since agents do not suffer from money illusion.

This is the new key equation. It captures the demand for goods given changes in their relative prices. Because that choice is static, no terminal condition is needed. The policy tool is the central bank's choice of the good (or goods) to which to peg the unit of account.

6.1 Commodity pegs

The central bank can choose any arbitrary good, say good 0, to denominate its reserves. This can happen by decree: the central bank simply announces that 100 dollars of reserves will be able to buy one gram of gold. It can then issue reserves (which recall it can do in unlimited amounts) to buy and sell gold to keep this exchange rate fixed forever.

This uniquely determines the price level. From equation (52), having defined that $p_t(0) = 1$, the price level p_t is determined. Relative price movements would however lead the price level to deviate from target.

The central bank would like to adjust the peg to relative-price movements to improve its effectiveness. However, it only has estimates of these. The most effective rule is: $p_t(0) = p_t^* + \sum_{i=1}^{I} \omega_i \hat{\rho}_t(i)$. Its effectiveness is:

$$\varepsilon_t = \sum_{i=1}^{I} \omega_i(\rho_t(i) - \hat{\rho}_t(i)), \tag{53}$$

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Changes in the supply of good 0, or in the public's taste for it, become sources of deviations of inflation from target. Moreover, if good 0 is a complement with others in consumption, then the impact on relative prices across all goods can be large. The ideal commodity to peg the price level to has a stable supply and is not complementary or substitutable with many other goods. Gold or other precious metals meet these two criteria and this is why they have often been used with this approach. Still, relative-price movements are large enough that commodity pegs have tended to generate large ε_t.[41]

6.2 A peg to a basket

Instead of picking a good, the central bank could choose a wide consumption basket to which to peg reserves. In this case, it would have to come up with estimates of what the weight of each item in this consumption basket is in any given period $\hat{\omega}_{i,t}$. In this case, the most effective policy rule now is:

$$\sum_{i=0}^{I} \hat{\omega}_{i,t} p_t(i) = p_t^*, \tag{54}$$

and the control error is: $\varepsilon_t = \sum_{i=0}^{I} (\omega_{i,t} - \hat{\omega}_i) p_t(i)$. While this approach is more demanding of the central bank and harder to implement, it may lead to smaller errors.

6.3 An exchange-rate peg

Nowadays, more common than pegging to gold is pegging to a foreign currency. This is especially true for small open economies, which import goods from other countries, often denominated in a dominant foreign currency. By accumulating a large amount of this currency, the central bank can stand ready to buy or sell it against its domestic reserves to maintain the peg.

Assume that aside from $I + 1$ domestic goods, the economy also imports $J + 1$ foreign goods, each with a foreign price $p_t(j)$ (in logs). The exchange rate (also in logs) between the domestic and the foreign units of account is q_t. Letting α denote the measure of home bias, the domestic price level is then equal to:

$$p_t = \alpha \sum_{i=0}^{I} \omega_i p_t(i) + (1 - \alpha) \sum_{j=0}^{J} \omega_j (p_t(j) + q_t) = \alpha \sum_{i=0}^{I} \omega_i p_t(i) + (1 - \alpha)(p_t^F + q_t) \tag{55}$$

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[41]See Bordo (2005) for a discussion.

where p_t^F is the price index of the imported goods in foreign currency.

Now, the optimality condition between any two domestic and foreign goods is: $\rho_t(i,j) = p_t(i) - p_t(j) - q_t$, where $\rho_t(i,j)$ is the marginal rate of substitution between consumption of domestic good i and foreign good j. It then follows that: $\sum_{j=0}^{J} \omega_j \rho_t(i,j) = p_t(i) - p_t^F - q_t$. Replacing for $p_t(i)$ in equation (55) delivers:

$$p_t = q_t + p_t^F + \alpha \sum_{i=0}^{I} \sum_{j=0}^{J} \omega_i \omega_j \rho_t(i,j). \tag{56}$$

The second and third term on the right-hand side are exogenous with respect to the price level. An exchange-rate target peg is a choice of q_t. Thus, it uniquely pins down the price level. Different policy rules for q_t have names like crawling pegs or managed floats, and they lead to different price levels, closer or further from the target.

One common reason why this approach for controlling inflation has failed when implemented is that the choice of the q_t target is not made with the goal of delivering a target price level. In particular, often pegs imply a fixed q_t over time. Changes in $\rho_t(i,j)$ and consequently in the real exchange rate then lead to wide fluctuations in p_t.[42]

Another reason is that while central banks announce a target for q_t, they do not commit to it, and deviate when the implications for real activity or foreign trade become unpleasant. This paper does not cover the difficulty of controlling inflation when the central banks cannot commit to a monetary policy approach, or to a rule within this approach, but this can be important in practice.[43]

7 The real activity approach: Phillips curves

Among policymakers, there is a strong tradition of thinking of monetary policy as affecting real activity. If resources are used more intensively in the economy, this will raise the marginal costs of production, encouraging firms to raise their relative prices. As they all do so, the absolute price level rises, and inflation results. The central bank's approach to controlling inflation combines its ability to affect real activity, and a Phillips curve linking real activity to inflation.

So far, we have used (perhaps excessively) the classical dichotomy assumption that separates the control of inflation from the effects of monetary policy on the real economy.

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[42]See Obstfeld and Rogoff (1995) or Ilzetzki, Reinhart and Rogoff (2019).
[43]For instance, see Frankel (2010).

In this section, we let go of the assumption that output is exogenous. We replace it with the classic assumption in the new Keynesian literature that monopolistic firms choose the price of the good they produce to increase proportionally with marginal costs and a markup. The price they choose is subject to nominal rigidities, and firms stand ready to produce whatever is demanded at that price. As a result, inflation depends on firms' expectations of what they thought inflation would be when they were last able to choose their price, as well as on what their actual marginal costs are. Nominal rigidities and demand-determined output delivers an expectations-augmented Phillips curve (in log-linearized variables):

$$\pi_t = \pi_t^e + \kappa w_t + z_t. \tag{57}$$

There are three terms on the right-hand side. The last one, z_t, is a stationary mean-zero markup shock, capturing changes in the market power of firms. As usual, we take these disturbances to be exogenous with respect to the price level, as well as to policy, so they become a new source of shocks that the central bank must monitor.

The first term on the right-hand side, π_t^e, is a measure of expected inflation by the firms who set prices. Different models of nominal rigidities lead to different specifications of this term, and we will discuss a few alternatives in section 7.3.

The second term is a measure of aggregate real marginal costs w_t. The association with the level of slack in product and labor markets can significantly differ, and it is sometimes referred to as the real rigidities in the economy. In turn, the mechanism through which a central bank tool is able to affect slack and real marginal costs is central to the Phillips-curve approach. Section 7.1 discusses credit policies as the tool that achieves this, while section 7.2 considers other alternatives.

7.1 The New Keynesian Phillips curve and credit tools

The most popular model of nominal rigidities is the sticky price model of Calvo (1983). It assumes that firms can reset their price in a given period according to a Poisson arrival process. Firms are forward looking so, when choosing their price, they take into account the probability that they may not be able to change it in the future. Expected future inflation becomes an aggregate sufficient statistic for how expectations of future changes in the price level, marginal costs, and markups will evolve. The result from Calvo (1983) is that $\pi^e = \beta \, \mathbb{E}_t(\pi_{t+1})$ where β is the steady-state discount factor.

One component of firms' marginal costs is the cost of having to raise funds externally

through loans to pay for investment and working capital. Let the lending rate on these loans be r_t^l. The opportunity cost of resorting to outside funding is then the external finance premium $r_t^l - r_t$, the difference between the cost of raising funds externally from banks and the opportunity cost of internal funds, which is the real interest rate. Marginal costs can then be written as: $w_t = \psi(r_t^l - r_t) + \zeta_t$, where ζ_t includes all the other stochastic determinants of marginal costs, which we take to be independent of inflation, and ψ is the elasticity of marginal costs with respect to the external finance premium.

Credit policies are another tool of the central bank it can use to control inflation. Central banks can have a direct effect on the price and amount of private credit to firms. They do so directly by setting the requirements for minimum reserves that banks must hold at the central bank as a fraction of their deposits (or loans), by setting requirements that banks' credit cannot exceed a multiple of their net worth, or by imposing that a fraction of banks' assets be held in the form of liquid marketable securities instead of loans. Moreover, in the past decade, both the Bank of England and the European Central Bank lent funds to banks at favorable rates under the condition that these funds would then be used to provide loans to firms.[44] We capture this myriad of policies by assuming that the central bank targets the lending rate subject to mistakes, u_t^l, that are independent of inflation: $r_t^l = x_t^l + u_t^l$. In this case, r_t^l is not a policy tool itself, but rather an intermediate target that a whole host of tools try to target.

Combining these ingredients with the Phillips curve gives rise to the following equation:

$$\pi_t = \beta \, \mathbb{E}_t(\pi_{t+1}) + \kappa \psi(x_t^l + u_t^l - r_t) + \kappa \zeta_t + z_t. \tag{58}$$

This is a difference equation for inflation, just like the ones we saw in section 3. As with the feedback rules for interest rates, this equation delivers determinate inflation only if a terminal condition ensures that the deviations of inflation from their target are $O(-\log(\beta))$. In the next sub-section, once we let r_t be affected by inflation, the question of determinacy will become more interesting.

Using the most effective policy rule for x_t^{l*}, the control errors are given by:

$$\varepsilon_t = \varepsilon_{t-1} + \sum_{s=0}^{\infty} \beta^s E_t \big[\beta(p_{t+1+s}^* - \hat{p}_{t+1+s}^*) + \kappa \psi(\hat{r}_{t+s} - r_{t+s})$$

$$+ (z_{t+s} - \hat{z}_{t+s}) + \kappa(\zeta_{t+s} - \hat{\zeta}_{t+s}) + \kappa \psi u_{t+s}^l \big]. \tag{59}$$

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[44]These policies were called the Funding to Lending scheme, and the Targeted Long-term Repurchase Operation, respectively.

A central bank that relies on the Phillips curve to control inflation is subject to three new sources of errors, beyond uncertainty about real interest rates and mis-communication about the target and the policy tool, as in previous analyses.

The first are errors on markup shocks, z_t which policymakers often refer to as "cost-push" shocks. They were famously used to explain the failure to control U.S. inflation in the 1970s. Separating changes in markups from movements in marginal costs is itself difficult in real time, and it often takes economists years or decades to agree on.

Second, there can be errors on what current and future marginal costs are, ζ_t. Productivity is notably volatile and hard to predict, as is the marginal cost of hiring an additional worker when there are heterogeneous firms and households. As a result, central banks end up employing significant resources to measure growth potential, output gaps, natural rates, and other concepts that go into marginal costs.

A third source of errors arises from the credit channel of monetary policy u_t^l. The source of the error is similar to the supply and demand shocks for currency in section 4. Financial innovation in credit markets or any shock to the banking sector will spill over to inflation if the central bank uses this policy tool.

Finally, the slope of the Phillips curve, captured by the parameter κ, determines the size of the impact of these errors on inflation. This parameter has been notoriously hard to accurately estimate, and it seems to have changed over time, leading to a new source of errors. Practical analyses of the relative success of central banks at controlling inflation often focus on shifts or mis-estimates of the size of κ.

7.2   Marginal cost as real activity and intermediate targets

When lending policies were used in the past as the main tool to control inflation, they often failed to deliver price stability.[45] Lending rates are a small component of marginal costs, so ζ_t is large. Likewise, the central bank's ability to affect lending conditions is imperfect, so u_t^l is also large.

Assume now instead that $w_t = \alpha(y_t - y_t^n)$, where y_t^n is the potential level of output, understood as the output level that would arise in the absence of nominal rigidities. The output gap, $y_t - y_t^n$, is then a measure of the level of slack in the economy. Among the macroeconomic variables that affect marginal costs, the level of output may well be the main driver through its effect on the relative scarcity of credit, the price of intermediate inputs, or the level of wages. Potential output y_t^n is exogenous with respect to the

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[45]These policies were famously associated with the Radcliffe report in the UK (Capie, 2010).

price level and plays a similar role to ζ_t in the previous section, but the amplitude of its fluctuations is arguably smaller.

Turning to central bank policy, we now drop the classical dichotomy assumption. Any of the previously discussed policy tools at the disposal of the central bank can potentially affect real activity and so marginal costs. The setting of interest rates, potentially combined with forward-guidance and going long policies, that were discussed in section 3, the printing of banknotes discussed in section 4, the quantitative and qualitative easing that vary the size and composition of the central bank's balance sheet in section 5, and the foreign exchange rate interventions discussed in section 6, all have effects on real activity and through the Phillips curve affect inflation as well, so any of them can be used to pursue price stability.

Central banks often choose *intermediate targets* for variables they do not control but which they can confidently steer using their menu of tools.[46] We consider the case where the central bank targets nominal income subject to shocks: $x_t^{py} = p_t + y_t + u_t^{py}$. Nominal income is a natural choice, both because the shocks u_t^n may be smaller than other variables, but especially because the Phillips curve captures the menu of possible combinations of p_t and y_t.[47] Making the sum of p_t and y_t the target for monetary policy in a Phillips-curve approach to controlling inflation is pedagogically natural and instructive.

Replacing the new policy tool into the new marginal cost equation and the new Keynesian Phillips curve from the previous sub-section gives one equation:

$$p_t - p_{t-1} = \beta \, \mathbb{E}_t(p_{t+1}) - \beta p_t + \kappa \alpha (x_t^{py} - p_t - u_t^{py} - y_t^n) + z_t \tag{60}$$

Rearranging gives a stochastic second-order difference equation for p_t:

$$\beta \, \mathbb{E}_t \, (p_{t+1}) - (1 + \beta + \kappa \alpha) p_t + p_{t-1} = \kappa \alpha (y_t^n + u_t^{py} - x_t^{py}) - z_t. \tag{61}$$

The quadratic expression $\beta - (1 + \beta + \kappa \alpha) x + x^2 = (1 - x\beta/\theta)(1 - \theta x)$ has two positive roots: $\theta < 1$ and β/θ. Therefore, the difference equation has the unique bounded solution:

$$p_t = \theta p_{t-1} + \beta \theta \sum_{j=0}^{\infty} (\beta \theta)^j \, \mathbb{E}_t \left[\kappa \alpha (x_{t+j}^{py} - y_{t+j}^n - u_{t+j}^{py}) + z_{t+j} \right]. \tag{62}$$

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[46]See Friedman (1990) for a discussion, and Svensson (2003) for a comparison of stating policy in terms of tools as opposed to targets for endogenous variables.

[47]A long literature studies the qualities of nominal income as an intermediate target with a focus on u_t^n: a classic is Bean (1983).

As usual, the equation by itself admits other solutions where the price level goes to infinity. Namely, one needs a justification for imposing:

$$\lim_{j\to\infty} (\beta\theta)^j \, \mathbb{E}_t(p_{t+j}) = 0. \tag{63}$$

However, unlike before, the classical dichotomy does not hold since, $p_t = x_t^{py} - y_t - u_t^{py}$. Policy chooses x_t^{py} to make sure that $y_t - y_t^n$ and u_t^{py} do not grow exponentially over time. Therefore, the condition in equation (63) becomes:

$$\lim_{j\to\infty} (\beta\theta)^j \, \mathbb{E}_t(y_{t+j} - y_{t+j}^n) = 0. \tag{64}$$

Unlike the price level p_t, output y_t is a real variable. Therefore, sometimes a transversality condition or a technological feasibility constraint will deliver the necessary terminal condition. In this case, explosive paths for inflation driven by self-fulfilling expectations come with explosive paths for real output that violate economic constraints, so they can be ruled out.

The most effective policy rule would track changes in markups. The effectiveness of the policy is given by:

$$\varepsilon_t = \beta\theta \sum_{j=0}^{\infty} (\beta\theta)^j \, \mathbb{E}_t \left[z_{t+j} - \hat{z}_{t+j} + \kappa\alpha \left(\hat{y}_{t+j}^n - y_{t+j}^n - u_{t+j}^{py} \right) \right]. \tag{65}$$

Compared with the direct lending policies, the trade-off is between the relative sizes of $\zeta_t - y_t^n$ versus $u_t^{py} - u_t^l$.

Because nominal income is a better predictor of marginal costs than the external finance premium, policy can lower the prediction errors by using income as an intermediate target. However, using an intermediate target instead of a direct tool introduces additional noise. This is a general lesson in the study of targeting rules for monetary policy. The closer is the target to the ultimate policy goal, the lower the errors in the relation between the target and the goal (at an extreme, the inflation control rule $p_t = p_t^*$ sets those to zero), but the larger the errors in using the tools to reach the target.[48]

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[48]For a discussion of targeting rules for inflation control, see Svensson (2003); Giannoni and Woodford (2017).

7.3 Alternative models of the Phillips curve and expected inflation

The literature has produced different models of nominal rigidities beyond sticky prices.[49] A leading alternative to sticky prices focuses on imperfect information as the reason why firms do not perfectly adjust their prices to shocks. A simple canonical model in this class has $\pi_t^e = \mathbb{E}_{t-1}(\pi_t)$, which results from some firms receiving information with a one-period delay, and so choosing their price based on their old information.[50] Another class of models assumes instead that agents are backward-looking and imitate econometricians in using past time-series data to make forecasts about the future. The simplest classic model is the adaptive expectations assumption that $\pi_t^e = \pi_{t-1}$.[51]

Starting with the imperfect-information case, and continuing to represent policy through nominal income targeting, the price level now is the solution to:

$$p_t = \mathbb{E}_{t-1}(p_t) + \kappa\alpha(x_t^{py} - p_t - y_t^n - u_t^{py}) + z_t. \tag{66}$$

To solve this equation, first take the public's expectations on both sides with respect to last-period's information. It then follows that $\mathbb{E}_{t-1}(p_t) = \mathbb{E}_{t-1}(x_t^{py} - y_t^n - u_t^{py} + z_t/\kappa\alpha)$. Replacing back into the equation gives the solution for the price level. Further replacing x_t^{py} with the most effective policy rule for nominal income gives the control errors under this approach:

$$\varepsilon_t = \frac{1}{1+\kappa\alpha}\left(\mathbb{E}_{t-1}(p_t) - \hat{\mathbb{E}}(p_t) + z_t - \hat{z}_t - \kappa\alpha(y_t^n - \hat{y}_t^n) - \kappa\alpha u_t^{py}\right). \tag{67}$$

Aside from markup shocks, mis-targeting of nominal income, and mis-measuring of potential output, there is a new source of deviations of inflation from target. The term $\hat{\mathbb{E}}(p_t)$ refers to the policymaker's forecast of the public's expectations of the price level. The central bank in this setting monitors surveys of expectations, market prices of assets that provide hedges against inflation, or any other piece of information that summarizes what people expected inflation to be. If these measures differ from the expectations rele-

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[49]Even within sticky price models, the Calvo model above is just the more popular, with several alternatives including adjustment-cost models, fixed-cost state-dependent adjustment models, partial indexation models, and others.

[50]Within imperfect information models, there are dynamic sticky information models, rational inattention models, models where agents receive imperfect signals on the state of the world, imperfect information arising due to higher-order uncertainty, and many combinations of these.

[51]Alternatives here include models with least-squares learning, extrapolative expectations, diagnostic expectations, and others.

vant for the Phillips curve, inflation will deviate from target.

Turning to the backward-looking Phillips curve, plugging $\pi_t^e = \pi_{t-1}$ and $w_t = \alpha(y_t - y_t^n)$ into equation (57) gives a second-order difference equation for the price level:

$$(1 + \kappa\alpha)p_t - 2p_{t-1} + p_{t-2} = \kappa\alpha(x_t^{py} - y_t^n - u_t^{py}) + z_t. \tag{68}$$

Unlike the other difference equations we have found, this is purely backward-looking. Given a set of initial conditions for prices in the past p_{-1} and p_{-2}, it pins down what p_t will be without any terminal condition. Inflation is determinate.

A policy rule that responds to past prices gives rise to control errors:

$$\varepsilon_t = \frac{1}{1 + \kappa\alpha}\left[z_t - \hat{z}_t - \kappa\alpha(y_t^n - \hat{y}_t^n) - \kappa\alpha u_t^{py}\right]. \tag{69}$$

The rule for the backward-looking model requires that x_t^{py} adjusts in response to past inflation. The derivation above assumed that past values of the price level are perfectly observed today. In reality, measurements are imperfect and measures of inflation are revised for many years after they are first published. The u_t^{py} in the expression above would include those future revisions of inflation in the past. From this view on policy, a major investment by the central bank should go into real-time measurement of inflation, complementing the measurement of marginal costs and markups.

7.4   The three-equation model

We now turn to the canonical three-equation New Keynesian model. It includes both the output-driven specification of real marginal costs and sticky prices à la Calvo (1983). We refer to the output gap as $\tilde{y}_t \equiv y_t - y_t^n$, so the Phillips curve is:

$$\pi_t = \beta\,\mathbb{E}_t(\pi_{t+1}) + \kappa\alpha\tilde{y}_t + z_t. \tag{70}$$

To this, the model adds a so-called IS relation:

$$\tilde{y}_t = \mathbb{E}_t(\tilde{y}_{t+1}) - (i_t - \mathbb{E}_t(\pi_{t+1}) - r_t^n). \tag{71}$$

This states that higher expected inflation increases current output through an increase in consumption as the incentive to save (the real interest rate) is reduced, other things

equal.[52]

The novelty relative to the previous analysis is that there is a two-way relationship between inflation and the real economy. The output gap affects real marginal costs and inflation in the Phillips curve. In turn, inflation affects the output gap in the IS curve. The new implication is that, if inflation is indeterminate, then so is output.[53] Nominal indeterminacy now implies multiple equilibria in terms of real outcomes, since for every possible inflation path, equations (70) and (71) give a corresponding output path.

One can complement the Phillips curve and IS curve with any of the approaches for monetary policy studied so far. Here we cover the case of interest rate rules, since they have attracted the most attention in the literature, but similar analyses can be done with any of the others.[54] Expanding the interest rate feedback rule to also include the output gap we get:

$$i_t = x_t^i + \phi \pi_t + \phi_y \tilde{y}_t. \tag{72}$$

The steps to solve for inflation are the same as in section 3 but now one must solve for output at the same time. The Phillips curve and IS curve can be combined with equation (72) to form the system:

$$\begin{pmatrix} \tilde{y}_t \\ \pi_t \end{pmatrix} = \frac{1}{1 + \phi_y + \kappa\alpha\phi} \begin{pmatrix} 1 & 1 - \beta\phi \\ \kappa\alpha & \kappa\alpha + \beta(1 + \phi_y) \end{pmatrix} \mathbb{E}_t \begin{pmatrix} \tilde{y}_{t+1} \\ \pi_{t+1} \end{pmatrix} + \frac{1}{1 + \phi_y + \kappa\alpha\phi} \begin{pmatrix} r_t^n - x_t^i \\ z_t \end{pmatrix} \tag{73}$$

A system of linear difference equations has a unique non-explosive solution if the number of eigenvalues of the matrix outside the unit circle is equal to the number of non-predetermined variables. In this case, both output and inflation can in principle jump, so both eigenvalues have to have modulus larger than 1. Standard linear algebra shows that this is the case if the following condition holds:

$$\phi > 1 - \frac{\phi_y(1 - \beta)}{\kappa\alpha}. \tag{74}$$

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[52]The IS results from combining the Euler equation, the Fisher condition, and the definition of the natural rate as the equilibrium interest rate without nominal rigidities. To derive it, start with the Euler equation (1) and assume log-utility for simplicity. Given the definition of the stochastic discount factor (5) and the Fisher equation (6), it follows that: $1/Y_t = \beta E_t \left[(1 + I_t(Y_{t+1}\Pi_{t+1}) \right]$. Log-linearizing and rewriting in terms of the deviations from output and the real interest rate that we derived in section 2, and which now denotes the natural levels y_t^n and r_t^n, gives equation (71).

[53]See Carlstrom and Fuerst (2002) and Nakajima and Polemarchakis (2005) for further discussion on sticky prices and real indeterminacy.

[54]See, for instance, the textbook treatment in Gali (2008).

This is a generalized version of the Taylor principle condition for determinacy from section 3. The same lesson carries through, together with the reliance on the same elusive terminal condition.

Likewise, the characterization of the effectiveness of the interest rate feedback rule as a monetary policy approach is also similar. The control errors are again a discounted sum of the public's perceived deviations between natural rates of interest, inflation targets, and markups over time, only with different weights. Finally, the analysis can be repeated with any of the other interest rate rules, the currency supply approach, or the solvency approach, without again changing the main conclusions.

In short, not much changes between different monetary policy approaches work when considering the two-way interaction between inflation and output.

7.5 The new Keynesian model at the effective lower bound

Price rigidities interacting with the effective lower bound have occupied a large strand of literature in the last decade. This is not the place to survey it, but instead we focus on how it affects the dynamics of inflation.[55] Consider an economy that is at the effective lower bound from period 0 to T. In section 3.6, we showed that: (i) any of the inflation approaches followed from date T onwards could determine P_T close to P_T^*, (ii) inflation before date T is negative and given by $P_t = (\beta\xi)^{t-T}P_T$, and (iii) there is a second equilibrium where the economy converges to $P_t = (\beta\xi)P_{t-1}$ and P_0 is indeterminate. We now revisit these conclusions in the presence of price rigidities.

The first of the three conclusions is the most straightforward, since it is unchanged. As earlier, if there is an interest rate feedback rule, the elusive terminal condition is what pins down inflation at date T. It is also still the case that different observed nominal interest rates paths from date T onwards do not pin down inflation by themselves, and that different paths of inflation from T onwards will lead to different paths of inflation before date T.[56]

The second conclusion, that there is deflation during the time when the ELB binds, is also still true. In particular, combine equations (70) and (71), and assume away all sources of shocks for simplicity ($y_t^n = z_t = 0$) to replace out output and get a second-

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[55]Eggertsson and Woodford (2003) are the classic analysis, and there is a large literature on fiscal multipliers at the ELB that we will not touch on.

[56]See Werning (2011) and Cochrane (2017).

order difference equation for inflation:

$$\pi_t = (1 + \beta - \kappa\alpha)\pi_{t+1} - \beta\pi_{t+2} - \kappa\alpha(i_t - r_t^n). \tag{75}$$

Now, during the period when the ELB binds $i_t = \ln(\xi)$. Since π_T and π_{T+1} are determined, there are two terminal conditions for this equation to give the whole path of inflation from 0 to $T - 1$. Just as in section 3.6, the central bank has no power to affect this path for inflation, which may be very far from the target inflation rate. During this path, deflation comes with output below its natural level (a recession).

A property of this system is that the larger is T, the lower is inflation and output at date 0. In the limit, a temporary interest rate peg that lasts forever has an unboundedly large effect on inflation and output today. Related, imagine that for a fixed number of periods $T^Z < T$, we have $r_t^n = \underline{r} < \ln\xi$, making it impossible to achieve a $\pi_t^* = 0$ target, but that between T^Z and T the central bank chooses to keep the nominal interest rate at $\ln\xi$ even though $r_t^n = 0$. The period between T^Z and T is a period of strict forward guidance: the central bank is keeping the nominal interest rate at the ELB even though it was not constrained by the state of the economy to do so. Then, the second-order difference equation above then has a startling property: the larger is T, keeping T^Z fixed (that is the larger is the period of forward guidance), the higher are inflation and output at date 0. In fact, if forward guidance is long enough, output may even go above the natural level at date 0. The combination of the ELB with the Calvo Phillips curve makes forward guidance in the distant future a powerful tool to control inflation in the present.

This has been called the forward guidance puzzle since it is easily contradicted by empirical estimates of the effects of forward guidance, and is more generally implausible.[57] At the same time, the literature has found that limits to rationality as discussed in section 3.5, incomplete insurance markets that change the IS relation in equation (71), or different models of price rigidity like sticky information that change the Phillips curve in equation (70) make the puzzle go away.[58]

Finally, we turn to the third and final lesson from section 3.6 on the presence of a

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[57] The puzzle was identified in Del Negro, Giannoni and Patterson (2012) and Carlstrom, Fuerst and Paustian (2015).

[58] Angeletos and Lian (2018), Gabaix (2019), and García-Schmidt and Woodford (2019) study deviations from perfect-foresight rationality in this context, Del Negro, Giannoni and Patterson (2012) and McKay, Nakamura and Steinsson (2016) explore incomplete insurance against income risks by households, and Carlstrom, Fuerst and Paustian (2015) explore sticky information price rigidities (see also Kiley (2016) and Eggertson and Garga (2019)).

permanent-deflation equilibrium that arises from a global analysis. This equilibrium remains with price rigidities. However, now small changes in how the nominal rigidities are modeled, including whether prices are sticky as in Calvo (1983) or as in Rotemberg (1982), or in how the sunspots that coordinate the equilibrium are introduced, or even in what numerical methods are used to characterize the global solution, seems to matter significantly for the properties of the equilibrium.[59]

8   Conclusion: a unified approach

Each section introduced one equation reflecting one economic mechanism that can drive inflation. Each of them suggested one or more policy tools that the central bank can use. For each, we presented policy rules that led to determinate inflation and that had a most effective version. For a refresher, table 2 collects all the approaches, mechanisms, tools, and rules.

Table 2: Map of different approaches fo price stability

Approach	Equilibrium condition / market / equation	Tools	Rules
No-arbitrage	Fisher equation / financial market / equation (6)	short-term nominal interest rate (on reserves or targeting bonds)	feedback rules: Taylor, Wicksell, core, inertial, ...
		real payment on reserves	fixed payment
		long-term nominal interest rate	feedback rules
			yield curve control
Monetarist	currency demand / market for currency / equation (31)	supply of banknotes	money-growth target
			monetarist with feedback
			seignorage
Solvency	CB budget constraint / CB solvency / equation (43)	CB net income	non-Ricardian surplus
			net worth
			fiscal dividend
Gold standard	price index / law of one price / equation (52)	peg to unit of account	commodity peg
			basket peg
			exchange-rate peg
Real activity	Phillips curve / firms' pricing / equation (57)	credit policies	lending rate target
		all of the above	nominal income target interest rate or all the above

All of the different economic forces co-exist in the dynamic general equilibrium. The sections in this paper, and different rows in the table, are mutually consistent. To see

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[59]See Fernández-Villaverde et al. (2015), Boneva, Braun and Waki (2016), and Christiano, Eichenbaum and Johannsen (2018) among others.

this, collect the main equation and policy rule from each section, expressing each in log-deviation terms for the sake of comparison.[60]

$$i_t = r_t + \mathbb{E}_t \, \Delta p_{t+1} \qquad \text{(Fisher)}$$

$$h_t - p_t = c_t - \eta i_t + u_t^d - u_t^s \qquad \text{(Currency demand)}$$

$$i_{t-1} - \pi_t + \beta^{-1} \left(A \hat{v}_t - \hat{V} a_t \right) = (1 - \beta) \sum_{j=0}^{\infty} \beta^j \, \mathbb{E}_t \left(m_{t,t+j} + s_{t+j} \right) \qquad \text{(Budget constraint)}$$

$$p_t = \alpha \sum_{i=0}^{I} \sum_{j=0}^{J} \omega_i \omega_j \rho_t(i,j) + q_t + p_t^F \qquad \text{(Price index)}$$

The system has four equations in five unknowns: $\{p_t, i_t, h_t, v_t, e_t\}_{t=0}^{\infty}$. Each policy approach is a choice of one of the policy rules. Adding just one of them closes the system and determines all variables. For example, if the central bank follows the no-arbitrage approach, a feedback rule and the Fisher equation pin down the price level and the nominal interest rate. Currency demand then solves for the amount of currency, the solvency condition determines how many reserves ought to be issued by the central bank, and the price index stipulates the nominal exchange rate.

Importantly, the central bank can choose only *one* policy approach. If more than one is chosen, mathematically the system is over-determined. Economically, the different policies are in conflict with each other and it is impossible for the policymaker to follow more than one at the same time.[61]

The literature has called the policy tool that is associated with the chosen policy approach the *active* one. The others are *passive*. So, if the central bank chooses an approach setting interest rates, then interest rate policy is active, while currency printing, the fiscal revenues of the central bank, and the exchange rate are all passive.[62]

Each policy approach came with a measure of effectiveness ε_t. When they are all

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[60]Note that i_t and $\hat{v}_t$ are the log deviations of $(1 + I_{t-1})$ and $\hat{V}_{t-1} \equiv V_{t-1}/P_{t-1}$ respectively. We omit the important terminal conditions, but each section already discussed them at length.

[61]It is possible that policy alternates between different policy approaches according to time and circumstances. We already saw how escape clauses can affect the dynamics of inflation. Some studies have found that different periods in US history were characterized by switches in the policy approaches (Bernanke and Mihov, 1998; Davig and Leeper, 2006; Bianchi and Ilut, 2017). In this case, the conditions for determinacy can change becoming themselves hybrids that depend on the frequency of policy shifts (Davig and Leeper, 2007; Farmer, Waggoner and Zha, 2009; Barthélemy and Marx, 2019).

[62]A different use of the word active and passive is to describe which of the two institutions, the central bank or the Treasury, is imposing its decisions on the other. If they are playing a game with each other, this will affect how the policy approach is chosen and set. Unfortunately, both definitions of active/passive are used in the literature, generating confusion.

brought together, this provides a way to compare approaches that does not rely solely on whether one finds some assumptions more convincing than others, but can be backed with estimates of their effectiveness. Central banks can control inflation. The way they go about doing it can be more or less effective, but in ways that are measurable and can be made scientific.

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